PHILADELPHIA UNIVERSITY
FACULTY OF ENGINEERING
Department of Electrical Engineering

Course Title: Power Systems Reliability (610586)

Prerequisite: Power Systems (1). (610481)

Text Books: Reliability Evaluation of Power Systems By: R.Billinton and R.N.Allan
Publisher: Kluwer Academic Publishers 2nd Edition – 2001-ISBN-13-9780306452598

Semester: First 2013/2014

Instructor: Dr. Mohammed Tawfeeq

Course Goals:

Introduce the student to the theory, techniques, and application of reliability engineering for electrical power system.

Time Schedule:

Duration: 16 Weeks **Lectures:** 3hours / week

Objectives:

1. Understand the principles of reliability engineering.
2. Learn statistical distributions as applied to power system components, circuit topologies in reliability consideration, redundancy techniques, and applications.
3. Learn analysis and aspects of distribution and transmission systems reliability evaluation.
4. Understand the reliability of generating systems and spinning reserve.
5. Learn the methods of reliability evaluation for composite systems.

Course Contents	Weeks
1- Introduction to Reliability Theory.	1
2- Reliability Mathematics: Review of Theory of Probability, Theory of Sets, Binomial Distribution, Poisson distribution, Exponential Distribution, Failure Rate, MTTF and MTBF.	3
3- System Configuration: Series Reliability, Parallel Reliability, Series-Parallel Reliability, Redundancy.	2
	2
4- Distribution System Reliability: Single- Bus Bar Schemes , Double- Bus BarSchemes ,Double- Bus Bar Double- Breaker Schemes, Double- Bus Bar System with Bus Couplers ,Ring Systems.	2
5- Transmission System Reliability: Frequency and Duration Method, Probabilistic Approach, Markov Technique Applied to Transmission System.	2
6- Generating Capacity Reliability Evaluation: Non-Probabilistic Method, Probabilistic Method, Loss of Load Probability Technique, State Probabilities, Loss of Energy probability	3

7- Composite System Reliability Evaluation, Reliability of Combinations of Generators Transformers & Transmission Lines. 2

8- Spinning Generating Capacity Reliability. 1

Mode of Assessment

1. First exam: (20%)
2. Second exam (20%)
3. Quizzes, Reports, H. works, and/or Projects: (20%)
4. Final exam: (40%)

References

Reference Books:

1. M.L.Shooman " Probabilistic Reliability: An Engineering Approach". Publisher: McGraw-Hill.

2. P.D.T.O Conner "Practical Reliability Engineering". Publisher: John Wiley. New York -3rd Edition.

3. R.Billinton and R.N.Allan". Reliability Evaluation of Engineering systems: Concepts and Techniques". Pitman Books, 1983.

4. I. Bazovsky "Reliability Theory and Practice". Publisher: Prentice Hall.

5. W.Blischke and D.Murthy,"Reliability modeling, Prediction, and optimization ", John Wilely ,New York ,2000.

Power System Reliability

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2013-2014

Introduction to Power system Reliability

- Reliability is a term relates to the ability of a system to deliver electricity to all points of utilization within the accepted standard and desired amount

- Historically, a power system has been divided into three almost independent areas of operation as follows:

  1. Generation System: facilities for the generation of electricity from economical energy sources.

  2. Transmission System: transportation system to move large energy blocks from generation facilities to specific geographical areas.

  3. Distribution System: within a specific geographical area distribute the energy to individual consumers (e.g., residential, commercial, industrial, etc.).

- Power system reliability studies are categorised into two:

  1) System adequacy
  System adequacy: related to the existence of sufficient facilities within the system to satisfy the consumer load demand. These include the necessary generators, transmission and distribution facilities.

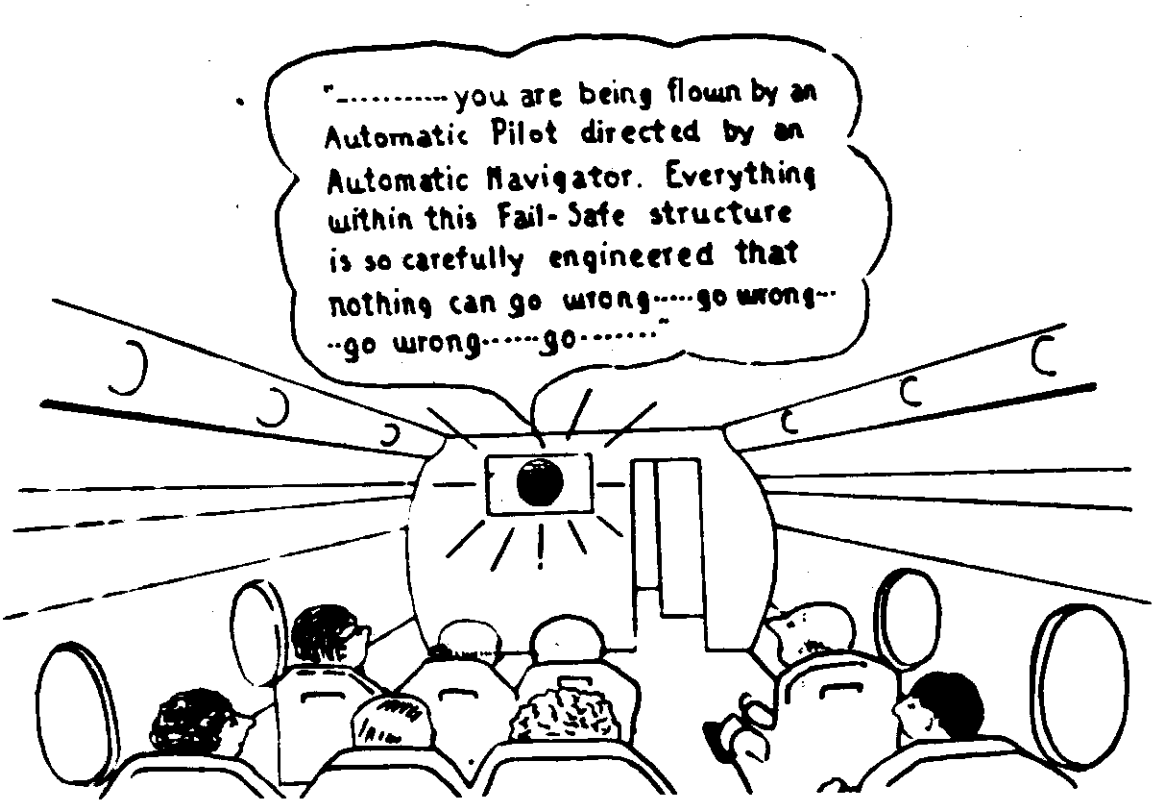
  2) System security
  System security: Related to the ability of the system to respond to disturbances. These include the minor and major disturbances resulting in dynamic, transient or voltage instability of power system.

  The reliability studies are mainly depending on the probability and statistics theories which are also use set theories. Hence in order to understand power system reliability the basic concepts of statistics, probability and set theory must be reviewed .

Introduction Reliability

- Reliability is a term meaning endurance, dependability, and good performance.

- For engineering systems, however reliability of a component or system is defined as the probability that this component or system is performing its function adequate under stated conditions for a specified period of time.

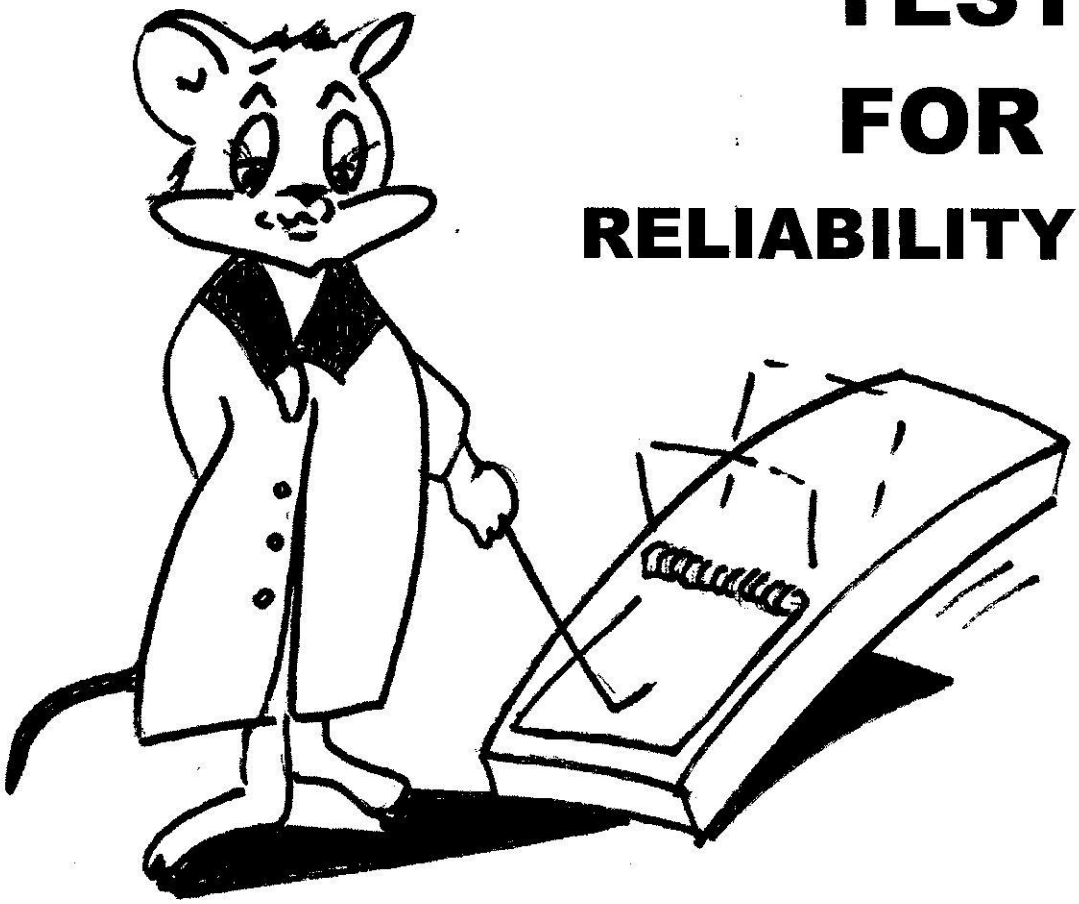

"..... you are being flown by an Automatic Pilot directed by an Automatic Navigator. Everything within this Fail-Safe structure is so carefully engineered that nothing can go wrong.....go wrong... ..go wrong.....go....."

The Importance of Humility

"It's only a random failure sir! it will never happen again

RELIABILITY	Dr. M.TAWFEEQ
INTRODUCTION	

TEST FOR RELIABILITY



ONE TEST *is Worth*

10,000 opinions

2.Basic Concepts of Set Theory

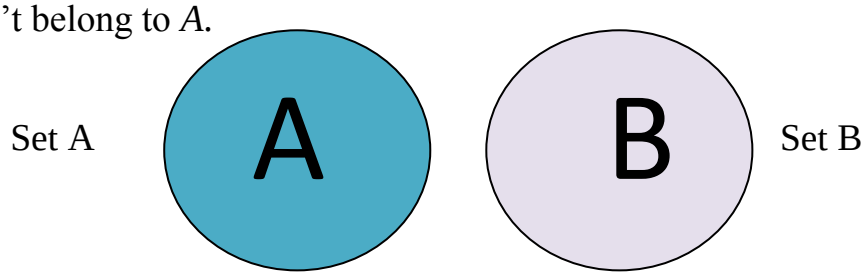
2.1. Sets and elements

Set theory is a basis of modern mathematics, and notions of set theory are used in all formal descriptions.

A ***set*** is a collection of objects which are called the *members* or *elements* of that set. If we have a set we say that some objects *belong* (or *do not belong*) to this set, *are* (or *are not*) *in* the set. We say also that sets *consist* of their elements.

Examples: the set of students in this room; the English alphabet may be viewed as the set of letters of the English language; the set of natural numbers ; etc.

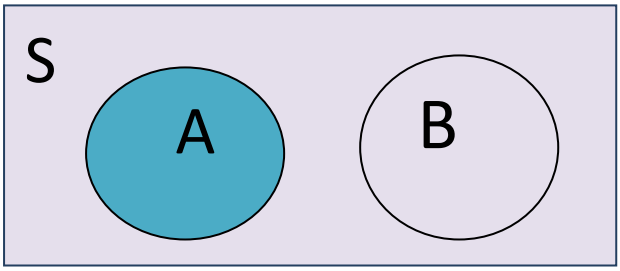
Sets can be *finite* or *infinite*.
There is exactly one set, the *empty set*, or *null set*, which has no members at all. A set with only one member is called a *singleton* or a *singleton set*. ("*Singleton of a"*)
Notation: *A, B, C, …* for sets; *a, b, c, …* or *x, y, z, …* for members.
$b \in A$ if *b* belongs to *A* (*B* ∈ *A* if both *A* and *B* are sets and *B* is a member of *A*) and $c \notin A$, if *c* doesn't belong to *A*.



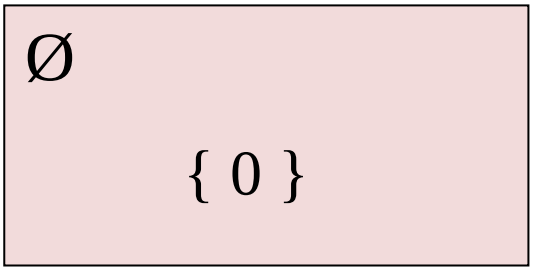
Sample Space ,(S) and Empty Set:

Also called a Universal Set

Ø is used for the empty set. Hence, Ø = {0}.



Universal set



Empty set

2.2. Subsets

A set *A* is a *subset of a set B* if every element of *A* is also an element of *B*. Such a relation between sets is denoted by $A \subseteq B$. If $A \subseteq B$ and $A \neq B$ we call *A* a proper subset of *B* and write $A \subset B$. (Caution: sometimes $\subset$ is used the way we are using $\subseteq$.)

Both signs can be negated using the slash / through the sign.

Examples:

$\{a,b\} \subseteq \{d,a,b,e\}$ and $\{a,b\} \subset \{d,a,b,e\}$.

A and B are subsets of $A \cup B$

2.3. Operations on sets: union, intersection.

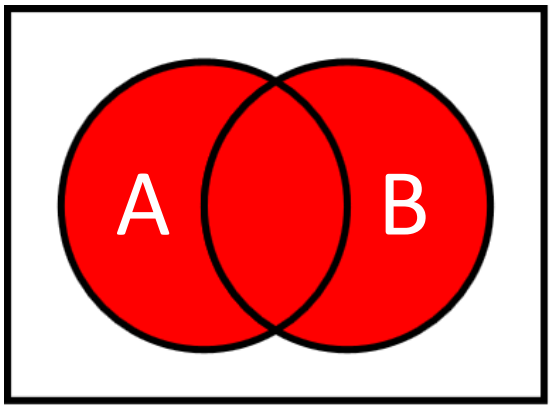
We define several operations on sets. Let *A* and *B* be arbitrary sets.

- The *union* of *A* and *B*, written $A \cup B$, is the set whose elements are just the elements of *A* or *B* or of both. In the predicate notation the definition is

$$A \cup B =_{\text{def}} \{ x \mid x \in A \text{ or } x \in B \}$$

Union of two sets

$A \cup B$



Commutative Law:

$A \cup B = B \cup A$ (like a+b =b+a in ordinary algebra)

Associative Law :

$A \cup (B \cup C) = (A \cup B) \cup C$ (like a+ (b+c) = (a+b) + c in ordinary algebra)

Examples. Let $K = \{a,b\}$, $L = \{c,d\}$ and $M = \{b,d\}$, then

$K \cup L = \{a,b,c,d\}$

$K \cup M = \{a,b,d\}$

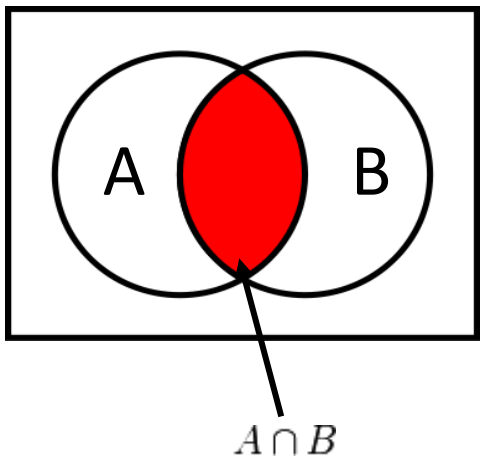
$L \cup M = \{b,c,d\}$

$(K \cup L) \cup M = K \cup (L \cup M) = \{a,b,c,d\}$
$K \cup K = K$
$K \cup \emptyset = \emptyset \cup K = K = \{a,b\}$.

- The *intersection* of *A* and *B*, written $A \cap B$, is the set whose elements are just the elements of both *A* and *B*. In the predicate notation the definition is

$$A \cap B =_{\text{def}} \{ x \mid x \in A \text{ and } x \in B \}$$

Intersection of two sets $A \cap B$



$A \cap B = B \cap A$
$(A \cap B) \subseteq A$
$(A \cap B) \subseteq B$

Examples:

$K \cap L = \emptyset$
$K \cap M = \{b\}$
$L \cap M = \{d\}$
$(K \cap L) \cap M = K \cap (L \cap M) = \emptyset$
$K \cap K = K$
$K \cap \emptyset = \emptyset \cap K = \emptyset$.

2.4 More operations on sets: difference, complement

Another binary operation on arbitrary sets is the *difference "A minus B"*, written $A - B$, which 'subtracts' from A all elements which are in B. [Also called *relative complement: the complement of B relative to A.*] The predicate notation defines this operation as follows:

$$A - B =_{\text{def}} \{ x \mid x \in A \text{ and } x \notin B \}$$

Examples: (using the previous *K, L, M*)
$K - L = \{a,b\}$
$K - M = \{a\}$
$L - M = \{c\}$
$K - K = \emptyset$
$K - \emptyset = K$
$\emptyset - K = \emptyset$.

2.5. Set-theoretic equalities

There are a number of general laws about sets which follow from the definitions of set- theoretic operations, subsets, etc. A useful selection of these is shown below. They are grouped under their traditional names. These equations below hold for any sets *X*, *Y*, *Z*:

1. *Idempotent Laws*

(a) $X \cup X = X$ (b) $X \cap X = X$

2. *Commutative Laws*

(a) $X \cup Y = Y \cup X$ (b) $X \cap Y = Y \cap X$

3. *Associative Laws*

(a) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
(b) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

4. *Distributive Laws*

(a) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

(b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

In ordinary Algebra :

(i) $a + (b \times c) \neq (a + b) \times (a + c)$

(ii) $a \times (b + c) = (a \times b) + (a \times c)$

5. *Identity Laws*

(a) $X \cup \emptyset = X$ (c) $X \cap \emptyset = \emptyset$

(b) $X \cup S = S$ (d) $X \cap S = X$

6. *Complement Laws* \

(a) $X \cup X' = S$ (c) $X \cap X' = \emptyset$

(b) $(X')' = X$ (d) $X - Y = X \cap Y'$

7. *DeMorgan's Laws*

(a) $(X \cup Y)' = X' \cap Y'$ (b) $(X \cap Y)' = X' \cup Y'$

8. Consistency Principle

(a) $X \subseteq Y$ if $X \cup Y = Y$ (b) $X \subseteq Y$ if $X \cap Y = X$

3. Probability Theory

Probability Concepts:

Probability is the science that deals with uncertainty .It is also called the measure of chance which means that the chance of an event to happen. It provides a mathematical definition of randomness.

Definitions:

- Statistical random experiment : Is the experiment in which its outcomes cannot be predicted with certainty. Or it is an empirical record of any phenomenon whose relative frequency of occurrence is uncertain.
- Outcomes: Is a particular result of the random experiment, denoted by small letters, a,b,c or x,y,z..).
- Sample space: All possible outcomes of given experiment. (Denoted by letter S).
- An event: A specific outcome or collection of outcomes; usually denoted by capital letters A,B,C or X,Y,Z)

The probability of an event is expressed on a scale from 0 to 1

✓ 0 means it will never happen
✓ 1 means that it is certain to happen

Hence probability of events will have values between 0 (failure) and 1 (success) except extreme cases.

– P (success) = Nos. of success / Nos. of possible outcomes; $p = s / (s+f)$
– P (failure) = Nos. of failures / Nos. of possible outcomes; $q = f / (s+f)$

Where; $p + q = 1$

The probability of an event is the number of times that a specific event occurs relative to the sum of all possible events that can occur. *This is the classical definition of the probability.*

Example 1 : The probability of rolling a 3 on a die is 1 out of 6, or 1/6.

Example 2 : Flipping a coin once can results in either a head (H) or a tail (T) ,i.e, 1 out of 2 , or 1/2. Here, S= {H,T}.

Example 3 : The probability of two coins flipped once coming up heads is 1 out of 4, or 1/4 .

- Relative frequency of an event : It is how often an event occurs relative to all other events that occurred in the experiment
- Certain event : the event will surely or certainly occurs . (S- exist) .
- Impossible (uncertain) event: The event cannot occur (S =ϕ).
- Mutually exclusive events :Two or more events which cannot happen together, .e.g. acquitted and sent to prison, the probability = 0.0
- Conditional probability The probability of event A happening, given that event B has already occurred, e.g. probability of going to prison (A) given that the offender was put on probation (B).

$$\text{This is symbolized} \quad P(A \mid B)$$

- Independent events: Two events A & B are considered independent if the conditional probability P (A | B) = P (A), e.g. probability of acquittal (A) given that it is raining outside (B).

Notes on Basic Probability Theory

It is possible to combine events to form new event using the operation of set theory.

1. Addition rule of probability

- Event (A∪B) is the union event and is defined as the event that occurs if A occurs or B occurs or both.
  - If the two events are not mutually exclusive :

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example 4 : What is the probability of drawing either a Jack or a Heart from a well shuffled deck of cards?

P (J or H) = P (J) + P (H) – P (J and H)

P (J or H) = P (4/52) + P (13/52) – P (1/52)

P (J or H) = (0.0769 + 0.2453) – (0.01923)

P (J or H) = 0.3077

- If the two events <u>are mutually exclusive</u> …

$$P(AUB) = P(A \text{ or } B) = P(A) + P(B)$$

<u>Example 5 :</u> What is the probability of drawing a Jack or a King?

P(J or H) = P(J) + P(H)

P(J or H) = P(4/52) + P(4/52)

P(J or H) = (0.0769 + 0.0769) = 0.1538

2. Multiplication Rule of Probability

Event **(**A∩B) is the intersection event and is defined as the event that occurs if A and B occurs, i.e., the probability of A and B happening together.

- The general rule

$$P(A \cap B) = P(A \text{ and } B) = P(B)\, P(A|B)$$

- If the events are independent of each other, this simplifies

$$P(A \cap B) = P(A)\, P(B)$$

Example 6: What is the probability of drawing a Jack of Hearts?

Since J and H are not mutually exclusive, therefore independent …

P (J and H) = P (J) P (H) = (4/52) (13/52)

= 0.01923

Notice that this is the same as

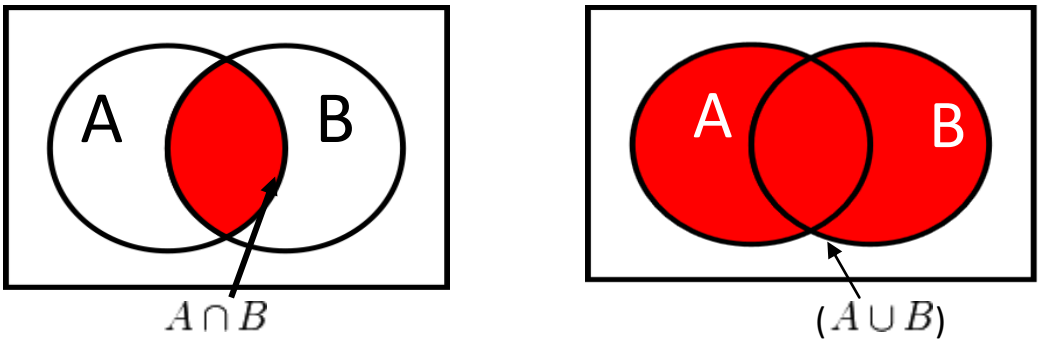
(1/52) = 0.01923

- Event A' (the complement of A) is the event that occurs if A does not occurs.

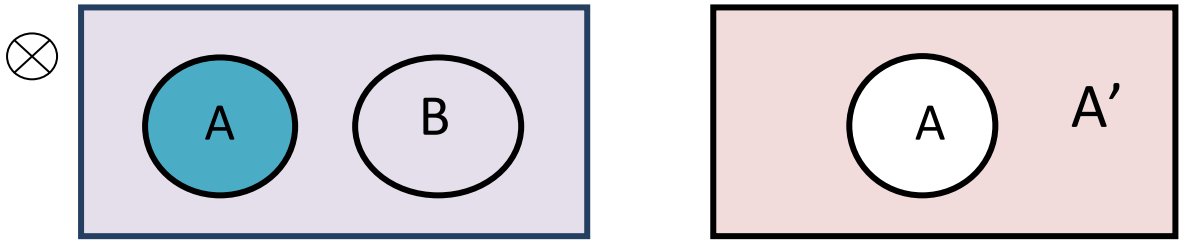
Venn diagrams

It is possible to use a Venn diagram to demonstrate the combinations of event . Probability theory can be mathematically expressed through set theory where Venn diagram is used to explain the set theory pictorially

- A Venn diagram is normally drawn as a rectangle which represents the total probability space S that encloses or represents the entire space being considered
- There may be two or more events such as A, B within this
- space S for which the probabilities must be combined



Venn diagrams for probability representation



Disjoint Complement

Venn diagrams for probability representation

⊗ As in the last Venn diagram A and B are disjoint. Hence, ($A \cap B$) has no elements then the elements A and B are said to be mutually exclusive. This implies that A and B cannot occur simultaneous.

Axioms:

1. If P (A) =1, then A is a certain event .
   If P (A) = 0 , then is an impossible event .
2. For an event A : $0 \leq P(A) \geq 1$ and P(A)'= 1- P(A).
3. P(S) = 1 always.
4. For any two events ,as in the second Venn diagram above,

$$P(AUB) = P(A) + P(B) - P(A \text{ and } B)$$

5. For two events A and B which are mutually exclusive:

$$P(AUB) = P(A) + P(B)$$

and P(A)'= 1- P(A). and AUA'=S , also : $P(A \cap B) = 0$.

6. The probability of two independent events both occurring is equal to the product of the individual probabilities.

$$P(A \cap B) = P(A) .P(B)$$

7. The probability of event A given that event B has occurred is equal to the probability of A and B both occurring divided by the probability of event B occurring. This is known as the conditional probability. No independence between A and B is assumed. In fact, rearranging the terms gives the probability of both events A and B occurring when A and B are not independent. Hence,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example : Now suppose 300 of the boys and 100 of the girls are interested in computer games. The school has 400 students out of 800 who like computer games. However, if a student is picked at random, the probability of finding a boy who is interested in computer games is not 400/800x400/800=0.25. It should be 300/800=0.375 from first principles.
The Product Rule does not apply here because the two events, being a boy and being interested in computer games, are not independent—boys seem to be more interested in computer games than girls. Instead, the conditional probability of the fourth theoremshould be used.

$$\begin{aligned} P(\text{boy} \cap \text{likes computer games}) &= P(\text{boy}|\text{computer games}) \\ &\quad \times P(\text{likes computer games}) \\ &= 300/400 \times 400/800 \\ &= 0.375 \end{aligned}$$

It does not matter which event is the dependent one and which event is the independent one. The results will be identical:

$$\begin{aligned} P(\text{likes computer games} \cap \text{boy}) &= P(\text{likes computer games}|\text{boy}) \times P(\text{boy}) \\ &= 300/400 \times 400/800 \\ &= 0.375 \end{aligned}$$

4. Permutation and Combination

Bernoulli Trials

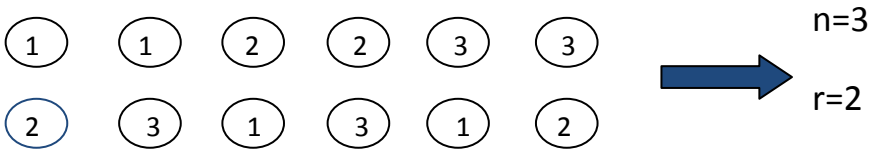
1. A coin is tossed once; the result is either Head (H) o Tail (T)

H or T

2. Two coins are tossed simultaneously (or a coin is tossed twice) the results are

H_1H_2, H_1T_2 , T_1H_2, or T_1T_2

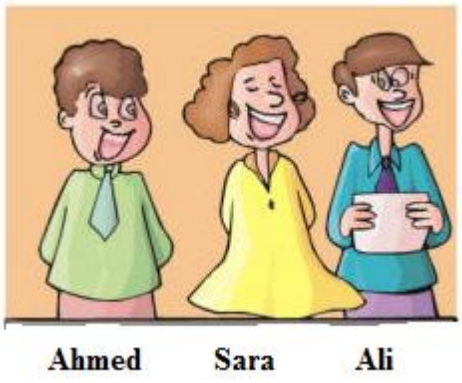
3. To generalize to n Bernoulli trials consider three object (n=3) and we want to arrange them 2 at a time (r=2) :

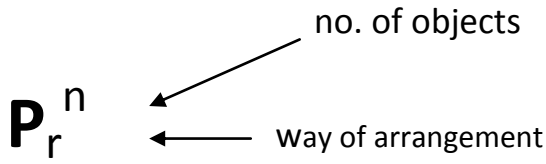


This leads to permutation.

!. Permutation : It is the sort of arrangement of things , or is the choice of *r* things from a set of *n* things without replacement and where the order is important . n objects we want to arrange them r at a time .

To a permutation, the trio of Ahmad, Sara and Ali is DIFFERENT from Ali, Ahmed and Sara. .

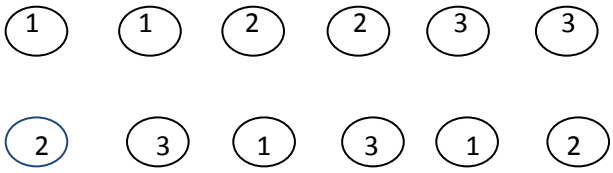




Mathematically it is written as:

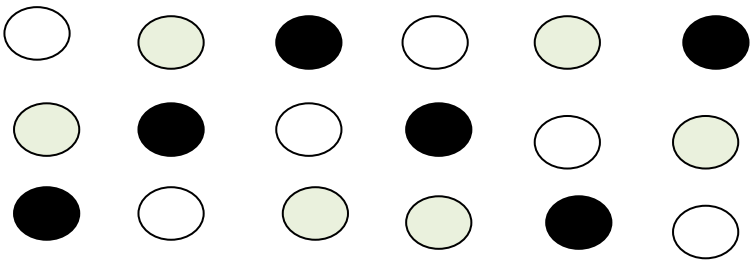
$$P_r^n = \frac{n!}{(n-r)!} \qquad \text{or} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

For the above example $P_2^3 = \frac{3!}{(3-2)!} = \frac{3x2x1}{1!} = 6$



For 3 object we want to arrange those 3 at a time;

$$P_3^3 = \frac{3!}{(3-3)!} = \frac{3x2x1}{0!} = 6$$



Example 1: What is the total number of possible 4-letter arrangements of the letters (m, a, t, h) if each letter is used only once in each arrangement?

$$\boldsymbol{P_4^4 =}\ {}_4P_4 = \frac{4!}{(4-4)!} = \frac{4\cdot3\cdot2\cdot1}{1} = 24$$

or simply $4!$

$${}_4P_4 = 4\cdot3\cdot2\cdot1 = 24$$

Example 2 : How many ways three books can be selected from seven books and placed in a shelf ?

There could be total of $\boldsymbol{P_3^7 = \frac{7!}{(7-3)!}}$ = 7 x 6 x 5 = 210 ways

2. Combination:

A **combination** is the choice of *r* things from a set of *n* things without replacement and where order does **not** matter (regardless the order of selection).

To a combination, the trio of Sara, Ahmed and Ali is THE SAME AS Ali, Sara and AHmed.

$$\boldsymbol{P_r^n = r!\, C_r^n}$$

Or $$C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

It is also can be written as :

$${}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Example 1:

Evaluate ${}_7C_2$:

$$ {}_7C_2 = \frac{7!}{2!(7-2)!} = \frac{7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}}{2\cdot1(\cancel{5\cdot4\cdot3\cdot2\cdot1})} = \frac{7\cdot6}{2\cdot1} = \frac{42}{2} = 21 $$

Example 2:

How many Combination of 2 letters can be made from the 6 letters : a,b,c,d,e,and f ?

Solution: Here n=6, r=2

$$ C_2^6 = \frac{6!}{2!\,(6-2)!} = 15 $$

Example 3:

There are 12 boys and 14 girls in Dr. Lazim reliability class. Find the number of ways Dr.Lazim can select a team of 3 students from the class to work on a group project. The team is to consist of 1 girl and 2 boys.

Solution:

Order, or position, is not important. Using the multiplication counting principle,

$$ \underbrace{{}_{12}C_2}_{boys} \cdot \underbrace{{}_{14}C_1}_{girls} = \frac{12\cdot11}{2\cdot1}\cdot\frac{14}{1} = 66\cdot14 = 924 $$

5.Probability distributions

Probability distribution A theoretical model that indicates the probability of specific events happening for a phenomenon distributed in a particular manner.
In statistics, numerous probability distributions are used to describe, explain, predict, and assist in decision making.

Examples include

- Binomial distribution
- Normal distribution
- Poisson distribution
- Lognormal distribution
- Exponential distribution
- Weibull distribution
- Gamma distribution
- Chi-square distribution

In this our course we shall consider only the binomial,poisson and exponential distributions.

1. Binomial Distribution

Binomial distribution is the one of the simplest discrete probability distribution. Consider a random trial having only two possible outcomes, such a trial is referred as a "Bernoulli trial" :
p = probability of success,
q= probability of failure
So that p+q =1

Consider flipping 3 coins once. What are the possible theoretical outcomes?

3 heads H1 H2 H3

3 combinations of 2 heads and 1 tail
H1 H2 T3
H1 T2 H3
T1 H2 H3

3 combinations of 1 head and 2 tails
H1 T2 T3
T1 H2 T3
T1 T2 H3

3 tails T1 T2 T3

Let the probability of a head = p, a tail = q, n = the number of coins flipped, and the probability of a head or tail = 0.5 (p=q=0.5)

This statistical experiment can be represented by the following binomial model:

$(p1 + q1)(p2+q2)(p+q3)$

If p1=p2=p3=p and q1=q2=q3=q

Hence $(p + q)(p + q)(p + q) = (p + q)^3$

$(p + q)^3 = 1p^3 + 3p^2q + 3pq^2 + 1q^3$

$(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$

This is for 3 cions, however for n coins;

$$(p + q)^n$$

Binomial with Unequal Probabilities

More generally, if p = probability of success and q = 1 – p = probability of failure, the probability of a specific sequence of outcomes where there are r successes and n-r failures is

$$p^r q^{n-r} = p^r (1-p)^{n-r}$$

Let X: random variable represent the total number of success out of n series Bernoulli trials.

r: the value that X may take (0,1,2,3,.....).

If we now want the probability of exactly X=r success out of n trials , then $p^r(1-p)^{n-r}$ must be by the number of ways that these r success can occur which is the number of combinations of n elements taken r at a time , given by $C_r^n = \frac{n!}{r!(n-r)!}$. Therefore the probability distribution function for the random variable X is

$$P_{X=r} = P[X = r, n, p] = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r} \qquad (1)$$

This equation can also be re-written as:

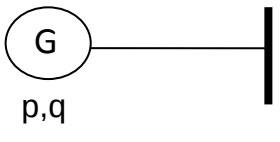
$$P_{X=r} = P[X = r, n, q] = \frac{n!}{r!(n-r)!} q^r (1-q)^{n-r} \qquad (2)$$

Meaning of p and q in power system component:

Consider a generating unit , if the unit is 10 hourrs during on year , then ,

$$P = \frac{8760-10}{8760} = 0.998858$$

$$q = \frac{10}{8760} = 0.001142$$



And p + q = $0.998858 + 0.001142 = 1$

Example 1 : consider two generators G1 AND G2 having probabilities of success (working) p1,p2 and failure (not working) q1,q2 respectively .The two generators are connected to the same bus bar to supply a common load, then;

Applying Binomial law:

P1+q1 =1 and p2+q2 =1

(P1+q1) x (p2+q2) =1

P1p2+p1q2+p2q1+q1q2=1

Neglecting maintenance period:

We have 4 cases:

P1p2 Two units on work

p1q2 Unit 1 working Unit 2 not working

p2q1 Unit 2 working Unit 1 not working

q1q2 Unit 1 and Unit 2 not working

Special case: If on unit can supply all the power to the load (design consideration) ,then

P_{system} = P1p2+p1q2+p2q1 (system available)

Q_{system} = q1 q2 (system unavailable)

If p1 = p2 = 0.99 , q1 = q2 = 0.01,

Q_{system} =q1q2 = (0,01) x (0.01) = 0.0001

P_{system} = 1- Q_{system} = 1- 0.0001= 0.9999

General Case :

If we have (n) non-identical generating units operating in parallel and supplying a common load, as shown in Fig.1

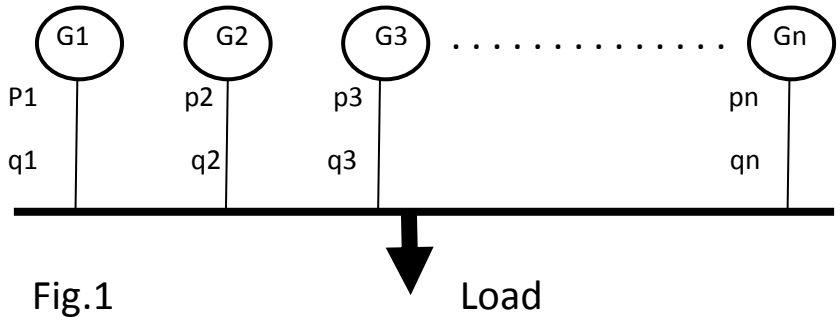


Fig.1

Here,

(P1+q1) x (p2+q2) x (P3+q3) x . . . . . .x (pn+qn) = 1

This is difficult to be evaluated , but if we assume that all units are identical , then

P1 = p2 = P3 =. . . . . .= pn = p

q1 =q2 = q3 =. . . . . .= qn =q

or $(p+q)^n =1$

and $(p+q)^n =\sum_{r=0}^{n} C_r^n \, p^r (1-p)^{n-r}$

Which represents a Binomial distribution

Example2:

An electric power system supplied from a total of 150 generating units. Calculate the probability of having (a) 2, (b) 5, and (c) 10 units being out of service simultaneous, given that the outage rate (q) of each unit is 0.06.

Solution:

(a) Applying equation 2 , the probability of having r= 2 units out of service is

$$P_{X=r} = P[X = r, n, q] = \frac{n!}{r!(n-r)!} q^r (1-q)^{n-r}$$

$$P[X = 2,150,0.06] = \frac{150!}{2!\,(150-2)!} 0.06^2 (1-0.06)^{150-2} = 0.00424$$

(b) Similarly the probability of having r =5 units out of service is

$$P[X = 5,150,0.06] = \frac{150!}{5!\,(150-5)!} 0.06^5 (1-0.06)^{150-5} = 0.05839$$

(c) the probability of having r = 10 units out of service is

$$P[X = 10,150,0.06] = \frac{150!}{10!\,(150-10)!} 0.06^{10} (1-0.06)^{150-10} = 0.12230$$

Note that the probability of having 10 units out is highest.

Conclusion:

An experiment must have four properties for the binomial distribution to be applicable:

1. Fixed number of trials
2. Each trial must result in success or failure.
3. All trials must have identical probabilities of success.
4. All trials must be independent.

6. The Probability Distribution

2. The Poisson distribution

Poisson distribution is a discrete probability distribution applied when the number of occurrence of an event in a given time is purely by chance. In this case Binomial distribution is not applicable although the rate of occurrence is fixed.

Example of such events are , the probability of a certain number of lightning strikes in a given time , the number of occurrence of fault in a long power cable and the probability that there a certain number of telephone calls in a given period of time.

With an infinite number of possible points for the random variable. The probability that the random variable will take on a value x =r is given by

$$P(X = r) = \frac{(\lambda t)^r}{r!} e^{-\lambda t}$$

where λ is a parameter describing the number of success per unit time (rate of occurrence of event in a time interval (t)) . The rate of occurrence λ is also called the failure rate if X represent the failure event.

Example1:

On a large power system, the average number of multi-core underground cable fault per year per 100 km of cable is 0.5 . Considering a specified piece cable 10 km long, what is the probabilities of 0,1,2,.....etc fault during a 40 years period?

Solution:

Assuming the number of faults to be valid for the cable in question and for a period of 40 years, the average number of faults λt is:

$$\lambda t = \frac{0.5x10x40}{100} = 2.0$$

$$P(X = r) = \frac{(\lambda t)^r}{r!} e^{-\lambda t} = \frac{(2.0)^r}{r!} e^{-2.0}$$

For r=0 , 1 , 2, 3 ,4 , 5 faults :

P(0) = 0.135
P(1) = 0.270
P(2) = 0.270
P(3) = 0.180
P(4) = 0.090
P(5) = (0.036)

These values could be represented as shown in Fig.1 which gives the histogram of the events.

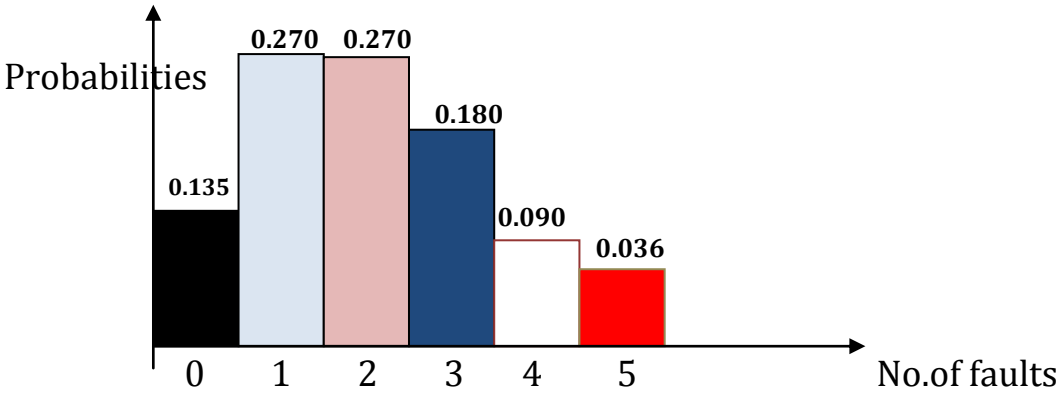


Fig.1

Example 2:
The failure of power transformers is assumed to follow Poisson probability distribution. Suppose on average, a transformer fails once every 5 years. What is the probability that it will not fail in the next 12 months? That it will fail once in the next 24 months?

Solution:

Failure rate λ = once in 5 years
=0.2 year
Number of expected failures in 12 months= 0.2.

Probability of having zero failures is given by

$$P(0) = \frac{(0.2)^0 e^{-0.2}}{0!}$$
$$= 0.8187$$

Number of expected failures in 24 months= 0.2x2=0.4.Probability of having exactly one failure in that period is

$$P(1) = \frac{(0.4)^1 e^{-0.4}}{1!}$$
$$= 0.2681$$

3. The Exponential Distribution

- How much time will elapse before an earthquake occurs in a given region?
- How long do we need to wait before a customer enters our shop?
- How long will a generating unit works without breaking down?
- How long will a transformer works without breaking down?

Questions such as these are often answered in probabilistic terms using the exponential distribution.

All these questions concern the time we need to wait before a given event occurs. If this waiting time is unknown, it is often appropriate to think of it as a random variable having an exponential distribution. Roughly speaking, the time we need to wait before an event occurs has an exponential distribution if the probability that the event occurs during a certain time interval is proportional to the length of that time interval.

Mathematically speaking, the exponential distribution is a continuous probability density function given by the formula

$$f(x) = \lambda\, \mathrm{e}^{-\lambda x}$$

where λ is a parameter of this probability function. It extends from 0 to ∞ and is illustrated in Fig. 2 or the Mean $E(X)$ = 1/ λ. The exponential distribution describes a probability that decreases exponentially with increasing x.

$$R(x) = \int_{x}^{\infty} \lambda\, \mathrm{e}^{-\lambda x}\, \mathrm{d}x = \mathrm{e}^{-\lambda x}$$

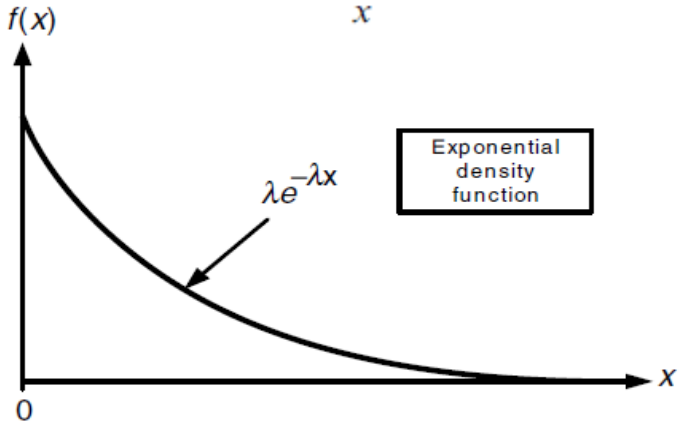


Figure 2. Exponential distribution.

The variation of the probability density function with the parameter λ is shown in Fig.3

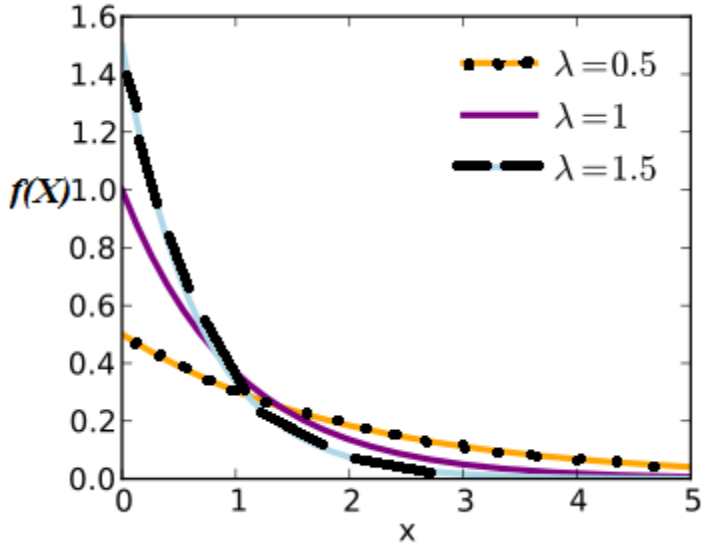


Fig.3

OTHER IMPORTANT PARAMETERS:

- **THE MEAN TIME TO FAILURES**

The expected average value for exponential distributed function may be considered as the average time for a failure to occur and is known as the mean time to failures or MTTF.
The expected value of a probability density function is given by

$$E(x) = \int xf(x)\,\mathrm{d}x$$

In our case, this becomes

$$\mathrm{MTTF} = \int_0^\infty t\lambda \mathrm{e}^{-\lambda t}\mathrm{d}t = 1/\lambda$$

It can be proven that the MTTF can also be obtained by integrating the reliability function over the entire range, that is,

$$\mathrm{MTTF} = \int_0^\infty R(t)\mathrm{d}t$$

This simplifies the calculation in most cases. For the simple exponential distribution, it becomes

$$\text{MTTF} = \int_0^{\infty} e^{-\lambda t} dt = 1/\lambda$$

Example 3:
There are 10 generators in a generating station. The units are assumed to have a failure rate of 0.02 per year. What is the mean time to failures in that station?
Solution:

$$\text{Mean time to failures} = \frac{1}{\lambda}$$

$$\lambda_{\text{station}} = 0.02 \times 10 = 0.2 \,\text{per}\, year$$

- The Variance and the Standard deviation

MTTF alone does not uniquely characterize a failure distribution . Other measures are necessary . one measure that is often used to further describe a failiare distribution is its variance σ^2 defined by

$$\sigma^2 = \int_0^{\infty} (t - MTTF)^2 \, f(t) dt$$

Upon Further simplification ,the above equation can be re-written as

$$\sigma^2 = \int_0^{\infty} (t^2 f(t) dt) - (MTTF)^2$$

The standard deviation of the exponential distribution is given by:

$$\sigma = \sqrt{\sigma^2}$$

The parameter λ , the mean time to failure $1/\lambda$, the variance σ^2 and the standard deviation σ all have significant physical meanings when the exponential distribution is applied to reliability assessments.

4. The Normal Distribution

Normal distribution is the most widely used probability distribution due to the fact that most things that are phenomena in nature tend to follow these distribution .Many things actually are normally distributed, or very close to it. For example, height and intelligence are approximately normally distributed; measurement errors also often have a normal distribution. It is a good approximation for many other distributions such as the binomial when the population is large. It is a continuous distribution; hence, the curve is the probability density function that takes on a symmetrical bell shape as illustrated in Fig. 4. The mathematical formula for the probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x-\mu)^2/2\sigma^2}$$

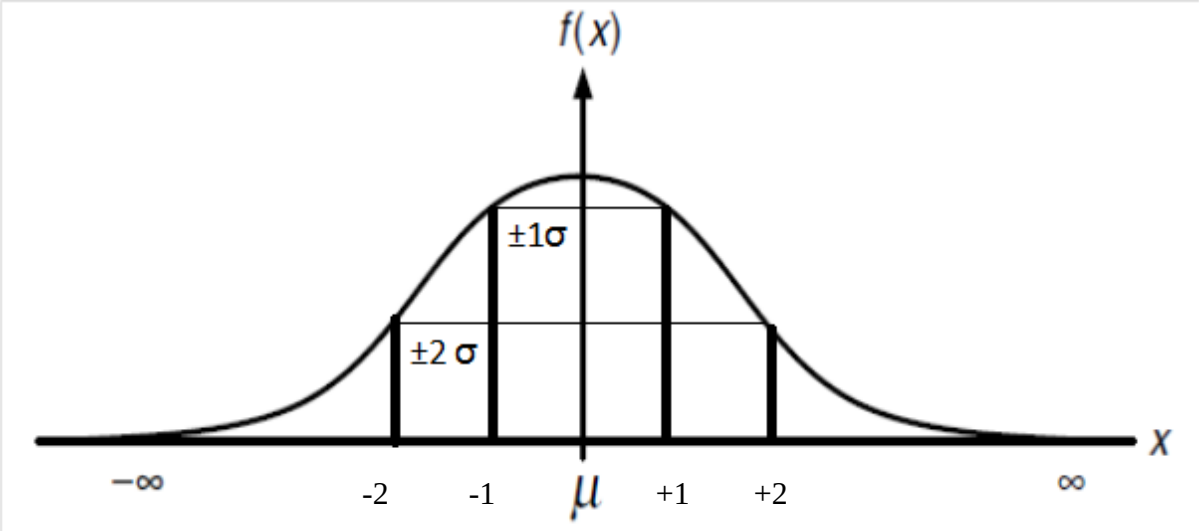


Figure 4. Normal probability density function.

Characteristics of the Normal distribution

- Symmetric, bell shaped
- Continuous for all values of X between -∞ and ∞ so that each conceivable interval of real numbers has a probability other than zero.
- $-\infty \leq X \leq \infty$
- Two parameters, μ and σ. Note that the normal distribution is actually a family of distributions, since μ and σ determine the shape of the distribution.

7. RELIABILITY PRINCIPLES

7.1 FAILURE RATE

Since every piece of equipment in a system will eventually fail if it is in service for a long period, there is a failure rate associated with each one. For some items, the failure rate is quite significant while for others it could be extremely low.

Failure rate is defined as the number of expected failures per unit in a given time interval. It is just an expected value. In calculating the failure rate of a group of units, the total operating time of the units should be used instead of the chronological time. The formula is

$$\text{Failure rate, } \lambda = \frac{\text{number of failures}}{\text{total operating time of units}}$$

Example 7.1

Ten transformers were tested for 500 h each and four transformers failed after the following test

time periods:

one failed after 50 h

one failed after 150 h

two failed after 400 h

What is the failure rate for these types of transformers?

Total operating time of units

$$= (1 \times 50 + 1 \times 150 + 2 \times 400 + 6 \times 500) \text{ unit h}$$
$$= 4000 \text{ unit h}$$
$$\lambda = \frac{4}{4000} = 0.001 \text{ failures/unit h}$$

Example 7.2

Thirty motors were tested for 200 h. Five motors failed during the test. The failures occurred after the following test times:

Motor 1 60 h

Motor 2 71 h

Motor 3 157 h

Motor 4 160 h

Motor 5 170 h
What is the estimated failure rate?
Solution:
Total number of unit operating hours

= 60+71þ+157+160+170+25 x 200
= 5618 unit h

$$\text{Failure rate, } \lambda = \frac{\text{number of failures}}{\text{total unit operating time}}$$

= 5/5168 unit h
= 0:00092 failures/h

7.2 Concept of Bathtub Curve

The life of a man has the following three major distinguishable periods as shown in Fig.7.1:
1. Infant mortality period
2. Useful life period
3. Wear-out period

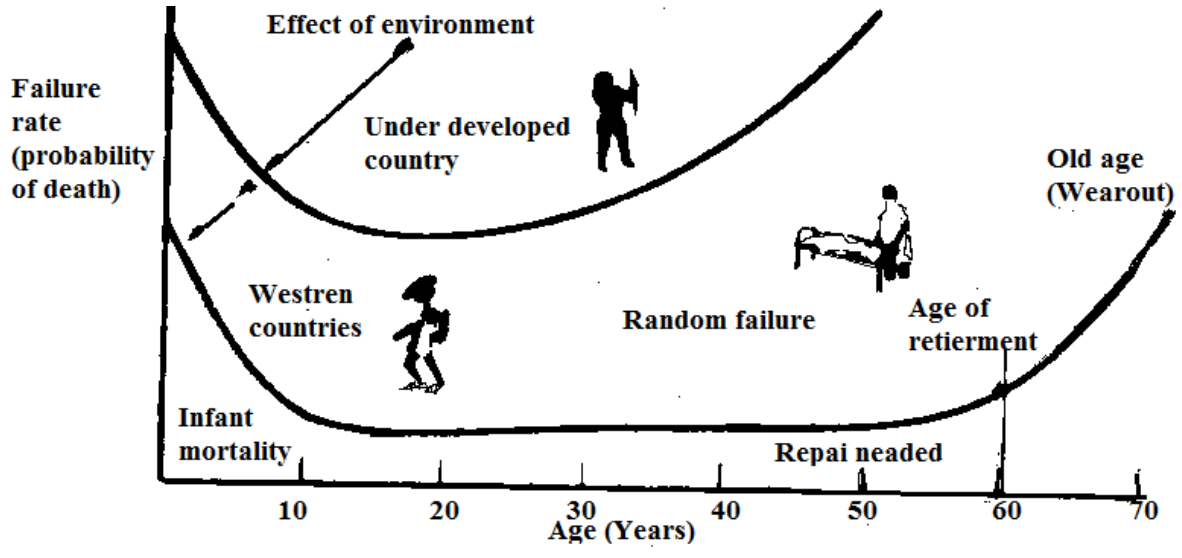


Fig.7.1 Bathtub curve failure rate versus time for human life.

Also the life of equipment has similar three major periods as shown in Fig.7.2:
1. Infant mortality period
2. Useful life period
3. Wear-out period

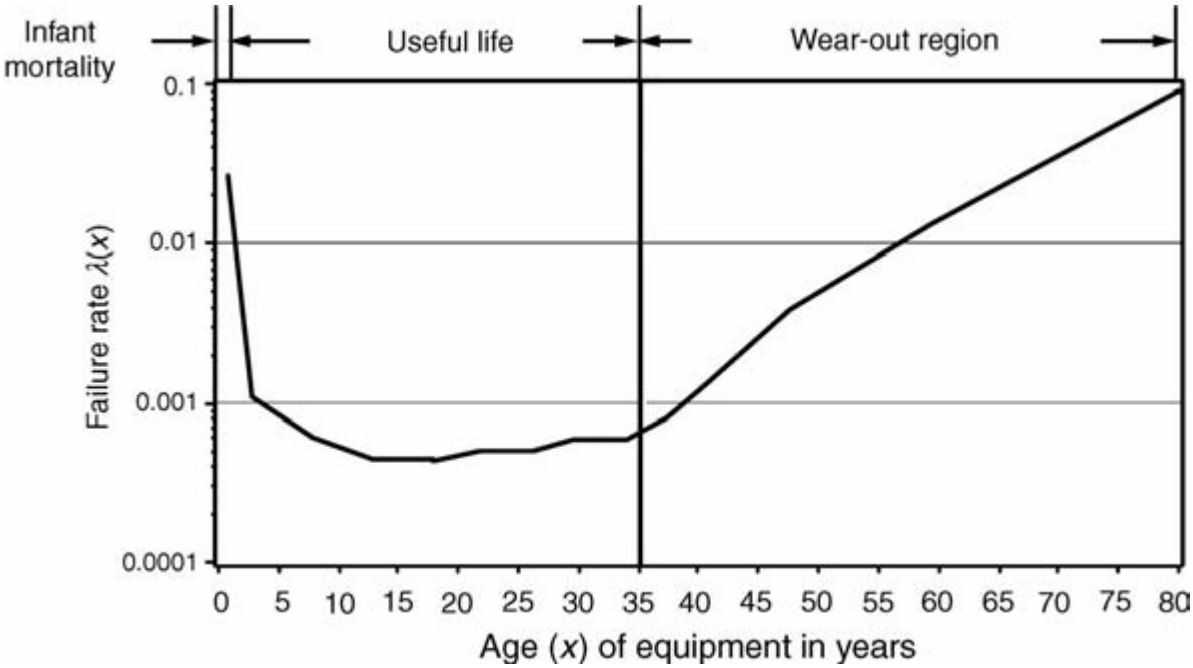


Fig.7.2. Bathtub curve failure rate versus time for components.

- In the infant mortality period, the failure rate is high due to the presence of weak spots from the manufacturing process such as poor workmanship, substandard components, and so on. As these weaknesses are manifested one by one by the stress of operation, the failure rate keeps decreasing until a low constant level is reached.
- The equipment then enters the useful life period where failures are due to chance and occur at random times. Failures in this period are also independent of the age of the equipment.
- When the components of the equipment start to wear out. From this time on, the failure rate rises rather rapidly due to deterioration.

Most reliability work deals with the useful life period when the failure rate is constant and the exponential distribution applies. one of the several possible distributions.

7.3 Reliability Model

Consider the case in which a fixed number of identical components (No) are tested.

Let ;

Ns (t) = number of component surviving at time t

N_f (t) = number of components failed at time t.

No = Ns (t) + N_f (t)

The reliability is the probability of not failing in a specified time interval. If the original population is No and N_f of them fail after time t, leaving Ns surviving, the reliability at time t which is the probability of surviving is

$$R(t) = \frac{N_s(t)}{N_o} = \frac{N_o - N_f(t)}{N_o}$$

$$R(t) = \frac{N_s}{N_o} = 1 - \frac{N_f}{N_o}$$

and

$$\frac{dR(t)}{dt} = \frac{-1}{N_o}\frac{dN_f}{dt}$$

The probability of failure is

$$Q(t) = \frac{N_f}{N_o}$$

Hence R (t) + Q (t) = 1 (They are complementary)

If we define $-\frac{1}{No}\frac{dNf}{dt}$ as the instantaneous failure density function

(=probability density function) *f(t)* , from which :

$$-\frac{dR(t)}{dt} = f(t)$$

By integrating both sides:

$$\int_{R(0)}^{R(t)} -dR(t) = \int_0^t f(t)dt$$

$$1- R(t) = \int_0^t f(t)dt \quad \text{.....} \quad (1) \qquad \text{(since R(0) =1)}$$

If the failure rate is constant, then we can define:

$$\frac{\mathrm{d}N_\mathrm{f}}{\mathrm{d}t} = \lambda N_\mathrm{s}$$

Since we have

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \frac{-1}{N_\mathrm{o}}\frac{\mathrm{d}N_\mathrm{f}}{\mathrm{d}t}$$

$$= -\lambda\frac{N_\mathrm{s}}{N_\mathrm{o}}$$

$$= -\lambda R(t)$$

$$\int \frac{1}{R}\mathrm{d}R = -\int \lambda\,\mathrm{d}t$$

$$\ln R(t) = -\lambda t$$

$$R(t) = \mathrm{e}^{-\lambda t}$$

$$Q(t) = 1 - R(t) = 1 - \mathrm{e}^{-\lambda t}$$

Example 7.3

Two hundred capacitors were installed and at the end of each year, the number of surviving units was tallied.

End of	Number of Units Remaining
Year 1	196
Year 2	188
Year 3	179
Year 4	175
Year 5	169

Based on these figures, what is the reliability of the capacitors for 5 years? The annual reliability of Year 4 ? Assuming the reliability function is exponential, that is, R (t) = e^{-t}, what is the failure rate for this formula?
Solution:
Reliability for 5 years = 169/200 = 0:845

$$\text{Annual reliability of Year 4} = \frac{\text{number of units surviving at the end of Year 4}}{\text{number of units at the beginning of Year 4}}$$
$$= 175/179$$
$$= 0.9777$$

$$R(t) = e^{-\lambda t}$$

$$\text{For 5 years, } R(t) = e^{-\lambda 5} \text{ (from formula)}$$
$$= 0.845 \text{ (from actual data)}$$

$$e^{-5\lambda} = 0.845$$
$$-5\lambda = ln(0.845)$$
$$= -0.1684$$
$$\lambda = \frac{-0.1684}{-5}$$
$$= 0.0337$$

Example 7.4
Ten thousand new oil circuit reclosers (OCRs) are put in service. They have a constant failure rate of 0.1 per year. How many units of the original 10,000 will still be in service after 10 years? How many of the original will fail in Year 10?
Solution:
Probability of survival is given by

$$R(t) = e^{-\lambda t}$$

In 10 years, probability of survival

$$R(10) = e^{-0.1\times 10} = e^{-1.0} = 0.3679$$

Out of 10,000 original units
10,000x0.3679=3679 should survive
Number of failures in Year 10

$$
\begin{aligned}
&= (\text{number of survivors after Year } 9) - (\text{number of survivors after Year } 10) \\
&= 10,000 \times e^{-0.1\times 9} - 3679 \\
&= 1000 \times e^{-0.9} - 3679 \\
&= 4066 - 3679 \\
&= 387
\end{aligned}
$$

8 . RELIABILITY OF COMPLEX SYSTEMS

8.1 Series Systems

Power system systems are made up of subsystems or components. Functionally, these subsystems or components are arranged in series or parallel connections or in combinations of the two.

For a series connection, every component is required to function for system success. Let us consider n component system X_1, X_2, X_3 ,....., X_n as shown in Fig 8.1.

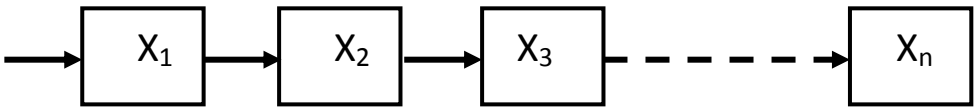


Fig. 8.1. A series connection.

The probability of success of the system is

$$P(S) = P(X_1 \cap X_2 \cap X_3 \cap \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \cap X_n)$$

$$or \quad P(S) = P(X_1).P(X_2).P(X_3)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots P(X_n)$$

Where $P(X_1)$ = Probability of success of X_1 = Reliability R_1(t)

$P(X_2)$ = Probability of success of X_2 = Reliability R_2(t)

$P(X_3)$ = Probability of success of X_3 = Reliability R_3(t)

. . . . . .

$P(X_n)$ = Probability of success of X_n = Reliability R_n(t)

$$R_{\text{system}}(t) = R_1(t) \times R_2(t) \times R_3(t) \times \cdots \times R_n(t)$$

$$or\ in\ general \qquad R_{syst}(t) = \prod_{i=1}^{n} R_i\,(t)$$

If the components have exponential failure probabilities with corresponding failure rates, λ1, λ2, λ3, and so on, then the system reliability

$$\begin{aligned} R_{\text{system}} &= \mathrm{e}^{-\lambda_1 t} \times \mathrm{e}^{-\lambda_2 t} \times \mathrm{e}^{-\lambda_3 t} \times \cdots \times \mathrm{e}^{-\lambda_n t} \\ &= \mathrm{e}^{-(\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n)t} \end{aligned}$$

The sum ($\lambda_1 + \lambda_2 + \lambda_3 + \text{-----} + \lambda_n$) is a constant = λ. It is the composite failure rate of the series system. From the system failure rate λ, the mean time between failures of the system can be calculated:

$$\text{MTTF}_{\text{system}} = 1/\lambda$$

8.2 Parallel Systems

A parallel system in a reliability sense means only one of the components in the parallel connections has to work in order that the system will function. Let us consider n component system X_1, X_2, X_3 ,....., X_n connected in parallel as shown in Fig 8.2.

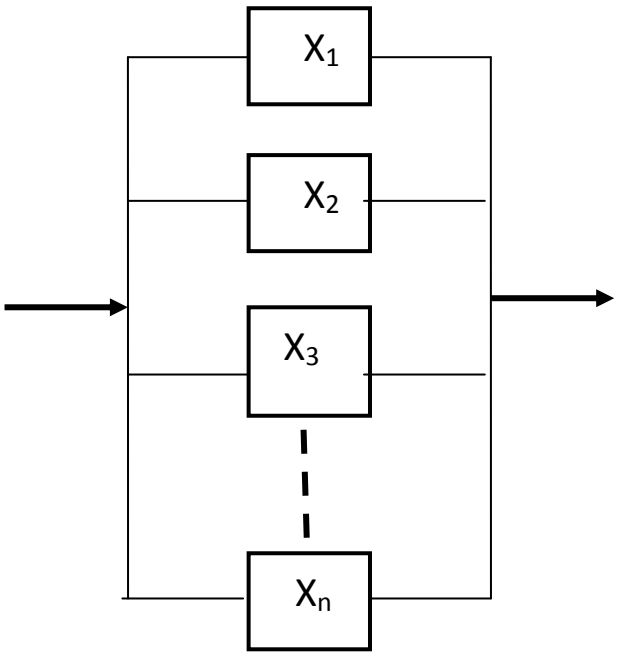


Fig. 8.2. A parallel connection

The probability of success of the system is

$$P(S) = P(X_1 \cup X_2 \cup X_3 \cup \dots\dots\dots\dots\dots\dots\dots\dots\dots \cup X_n)$$

$$P(f) = P(\overline{S})$$

$$P(f) = P(\overline{X_1} \cap \overline{X_2} \cap \overline{X_3} \cap \dots\dots\dots\dots\dots \cap \overline{X_n})$$

$$P(f) = P(\overline{X_1}).P(\overline{X_2}).P(\overline{X_3}) \dots\dots\dots\dots\dots\dots.P(\overline{X_n})$$

where $P(\overline{X_i})$ = Probability of failure of i^{th} component = $Q_i(t)$

Hence:

$$Q(t)_{\text{system}} = Q_1(t) \times Q_2(t) \times Q_3(t) \times \cdots \times Q_n(t)$$

$$\textit{or in general} \qquad Q_{syst}(t) = \prod_{i=1}^{n} Q_i\,(t)$$

If all the elements have the same Q(t)'s

$$\begin{aligned} Q(t)_{\text{system}} &= Q^n(t) \\ R(t)_{\text{system}} &= 1 - Q(t)_{\text{system}} \end{aligned}$$

If all the elements of the parallel system have constant failure rates and hence exponential reliability functions, as is usually assumed, the reliability of the parallel system will be

$$\begin{aligned} Q_{\text{system}}(t) &= Q(t)^n \\ R(t)_{\text{system}} &= 1 - Q(t)_{\text{system}} \\ &= 1 - (1 - e^{-\lambda_1 t}) \times (1 - e^{-\lambda_2 t}) \times (1 - e^{-\lambda_3 t}) \cdots (1 - e^{-\lambda_n t}) \end{aligned}$$

If all λ are the same,

$$R(t)_{\text{system}} = 1 - (1 - e^{-\lambda t})^n$$

Example8.1:
What is the reliability of the following system as shown in Fig. 8.3?

$$R(t)_{\text{A}} = R(t)_{\text{B}} = R(t)_{\text{C}} = 0.7$$

$$R(t)_{\text{D}} = 0.90$$

$$R(t)_{\text{E}} = 0.80$$

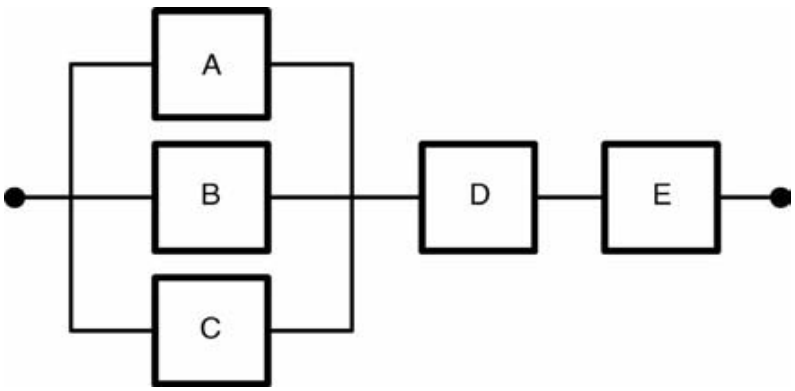


Fig.8.3 . A parallel–series connected system.

Solution:

Reliability of system

$$= R(t)_{A//B//C} \times R(t)_D \times R(t)_E$$
$$= (1 - Q(t)_A \times Q(t)_B \times Q(t)_C) \times R(t)_D \times R(t)_E)$$

$$Q(t)_A = Q(t)_B = Q(t)_C = 1 - R(t)_A$$
$$= 1 - 0.7$$
$$= 0.3$$

$$R(\text{system}) = (1 - 0.3 \times 0.3 \times 0.3) \times (0.90) \times (0.80) = 0.701$$

8.3 Availability Model

When a system fails, it will be out of service for some time until it is repaired or replaced. Even for systems with spare units, the system can be "down" if a failure occurs when no more spares are available. The percentage of time that the system is functioning is called the availability of the system. It is usually expressed as

$$\text{Availability} = A = \frac{\text{total hours of operation in 1 year}}{8760}$$

$$\text{Unavailability} = \bar{A} = \frac{\text{total hours of down time in 1 year}}{8760}$$

Since on average, it takes a time interval equal to the MTTF for the system to fail and a time interval equal to the MTTR for the system to be operational again, the availability, defined as uptime / (uptime+ downtime), can be expressed as

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

and unavailability can be expressed as

$$\bar{A} = \frac{\text{MTTR}}{\text{MTTF} + \text{MTTR}}$$

For systems that can be treated as a single component with a constant failure rate λ and repair rate μ, the availability

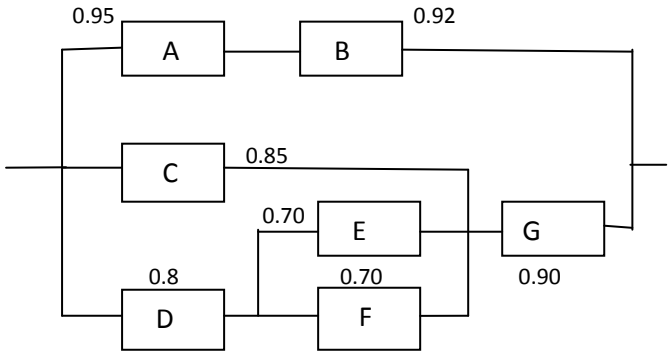
$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = \frac{1/\lambda}{1/\lambda + 1/\mu} = \frac{\mu}{\lambda + \mu}$$

and unavailability

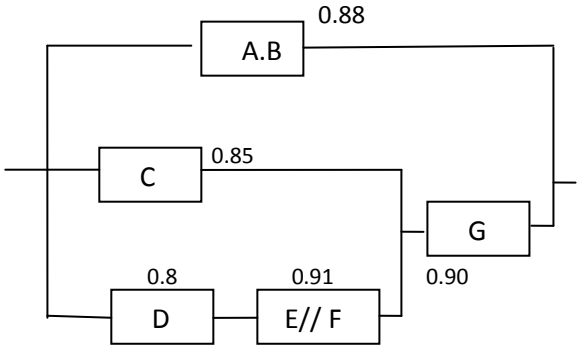
$$\bar{A} = \frac{\lambda}{\lambda + \mu}$$

9. Reliability of series parallel systems

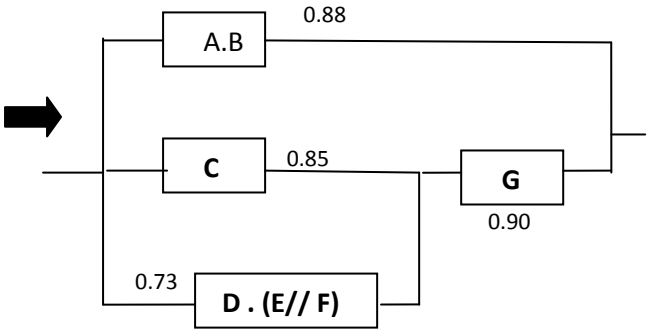
- In reliability evaluations, we reduce the complicated configurations by combining appropriate series and parallel branches of the model until a single equivalent element remains. This approach is called successive reduction method.
- For example, consider the structure shown below, the step of reduction are shown in the diagrams (a) to f) :



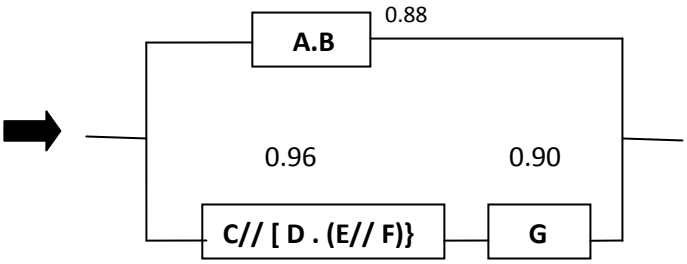
(a) System



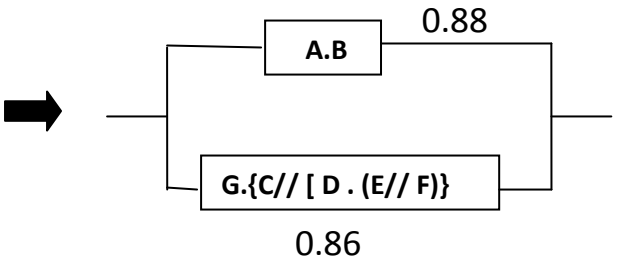
(b) First reduction



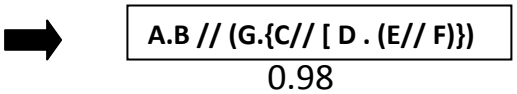
(c) Second reduction



(d) Third reduction



(e) Fourth reduction

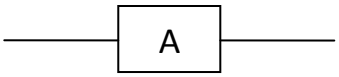


(f) Final reduction

Networks reliability evaluation using set theory

1. Single element network:

Consider a single element A



shown .For this element,

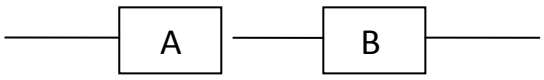
The probability of success P(S) = P(A) =P_a for which the element is **available** .

Also the probability of failure is P(f) =1- P(A) =Q_a for which the element is **unavailable**.

2. Two elements network:

(i) For two elements connected in series,

The availability of the network is



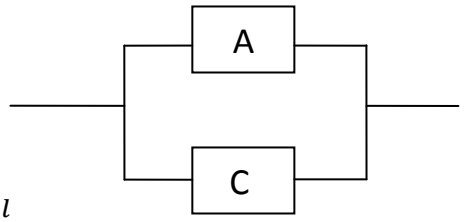
$$P(S) = P(A \cap B) = P_a \,.P_b = P_{series}$$

The unavailability of the network is

$$P(f) = P(\bar{A} \cup \overline{B}) = Q_a + Q_b - Q_a.Q_b = Q_{series}$$

(ii) For two elements connected in parallel,

The availability of the network is



$$P(S) = P(A \cup C) = P_a + \ P_c - P_a.P_c = P_{parallel}$$

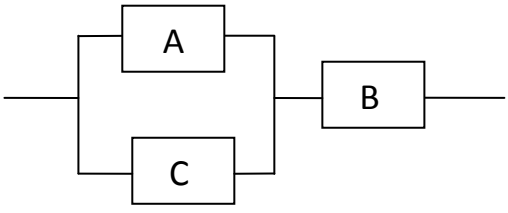
The unavailability of the network is

$$P(f) = P(\overline{A} \ \cap \ \overline{C}) = Q_a.Q_c = Q_{parallel}$$

2. Three elements network:

(i) Consider the case of three elements network as shown below:

The availability of the network is



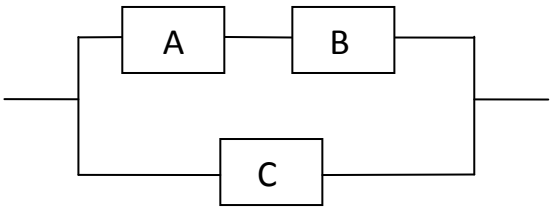
$$P(S) = P[(A \cup C) \cap B] = P[(A \cap B\ ) \cup (C \cup B)] = P_aP_b + P_cP_b - P_aP_bP_c$$

The unavailability of the network is

$$P(f) = P[\overline{(A \cup C) \cap B}\ ] = 1 - (P_aP_b + P_cP_b - P_aP_bP_c)$$

$$= 1 - P_aP_b - P_cP_b + P_aP_bP_c)$$

(i) If the same three elements network are re-connected as shown below:



The availability of the network is

$$P(S) = P[(A \cap B) \cup C] = P_aP_b + P_c - P_aP_bP_c$$

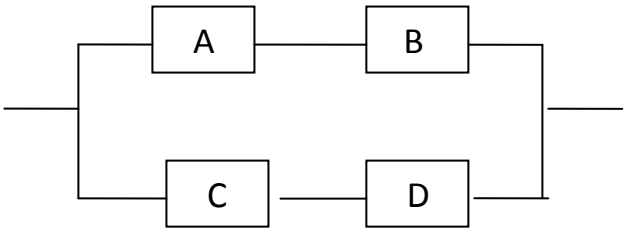
The unavailability of the network is

$$P(f) = P[\overline{(A \cap B) \cup C}\ ] = 1 - P_aP_b - P_c + P_aP_bP_c$$

3. Four elements network:

(i) Case when the four elements connected as shown below

The availability of the network is

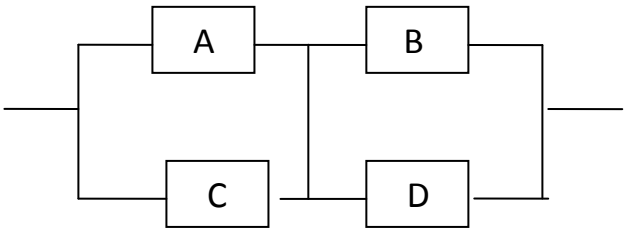


$$P(S) = P[(A \cap B) \cup (C \cap D)] = P_aP_b + P_cP_d - P_aP_bP_cP_d$$

The unavailability of the network is

$$P(f) = P[\overline{(A \cap B) \cup (C \cap D)}\ ] = 1 - P_aP_b - P_cP_d + P_aP_bP_c\,P_d$$

(ii) Case when the four elements connected as shown below



The availability of the network is

$$P(S) = P[(A \cup C) \cap (B \cup D)] = (P_a + P_c - P_a.P_c).(P_b + P_d - P_bP_d)$$
$$= P_aP_b + P_aP_d + P_cP_d - P_aP_bP_d + P_cP_b - P_aP_bP_d - P_aP_cP_d$$
$$- P_aP_bP_c + P_aP_bP_c\,P_d$$

The unavailability of the network is

$$P(f) = P[\overline{(A \cup C) \cap (B \cup D)}\ ]$$
$$= 1 - P_aP_b - P_aP_d - P_cP_d + P_aP_bP_d - P_cP_b + P_aP_bP_d + P_aP_cP_d$$
$$+ P_aP_bP_c - P_aP_bP_c\,P_d$$

10.Reliability evaluation of complex distribution systems

Reliability of power distribution schemes can always be improved at the expense of cost and size. The addition of extra paths proves to increase the continuity of supply. This is really where redundancy steps in, with its two constituents: components redundancy and unit's redundancy.

I. Single - Bus bar Scheme (System I)

The simplest and cheapest type of practical switchgear connection is the single bus bar scheme which consist of two isolators (A and C), and one circuit breaker (B). This scheme (System I) is shown in Fig. 1. The resultant system is available if the three elements are simultaneously in operation state, i.e

$$P_I = P(A \cap B \cap C) = P_a\, Pb\, Pc \qquad (1)$$

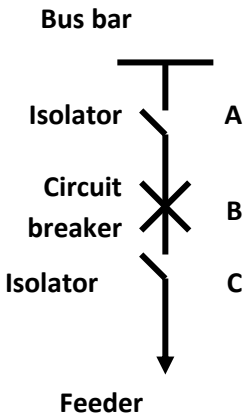


Fig.1 Single-Bus bar scheme

The unavailability of this scheme is given by

$$Q_I = 1 - P_I = 1 - P_a\, Pb\, Pc$$

$$= 1-(1-Q_a)(1-Q_b)(1-Qc)$$

$$= Q_a + Q_b + Qc - Qa\, Qb - Qa\, Qc - Qb\, Qc - Qa\, Qb\, Qc \qquad (2)$$

if the two isolators A and C are identical, then $P_a = Pc$ and $Qa = Qc$, thus

$$P_I = P_a^2\, Pb \qquad (3)$$

$$Q_I = 2\,Q_a + Q_b - Q_a^2 - 2\,Q_a\,Q_b + Q_a^2\,Q_b \qquad (4)$$

The yearly failure time of this system may be given by the following formula

$$T_{fI} = 8760\,Q_I \quad \text{Hours} \qquad (5)$$

From redundancy point of view this system has zero redundancy.

II. Double - Bus bar Double- Beaker Systems (System II)

The single bus bar layout does not allow for breaker or isolator maintenance without outage of supply. In order to overcome this difficulty, double- bus bar double- breaker schemes are used. However such a scheme is expensive and it is mainly used for large generating plants and critical loads where security of supply is paramount. Fig.2 shows the layout of this scheme (system II), where the system is available if at least one of the parallel paths A_1, B1, C1, or A_1, B2, C_2 is available. This can be obtained as follows:

$$P_{II} = P[\,(A_1 \cap B_1 \cap C_1) \cup (A_2 \cap B_2 \cap C_2)$$

$$= P\,(A_1 \cap B_1 \cap C_1) + P(A_2 \cap B_2 \cap C_2) - P\,(A_1 \cap B_1 \cap C_1)\,P\,(A_2 \cap B_2 \cap C_2)$$

$$= P_{a1}\,Pb_1\,Pc_1 + P_{a2}\,Pb_2\,Pc_2 - P_{a1}\,Pb_1\,Pc_1\,P_{a2}\,Pb_2\,Pc_2 \qquad (6)$$

The unavailability of this system is given by

$$Q_{II} = 1 - P_{II}$$

$$QII = 1 + Pa1\,Pb1\,Pc1\,Pa2\,Pb2\,Pc2 - Pa1\,Pb1\,Pc1 - Pa2\,Pb2\,Pc2 \qquad (7)$$

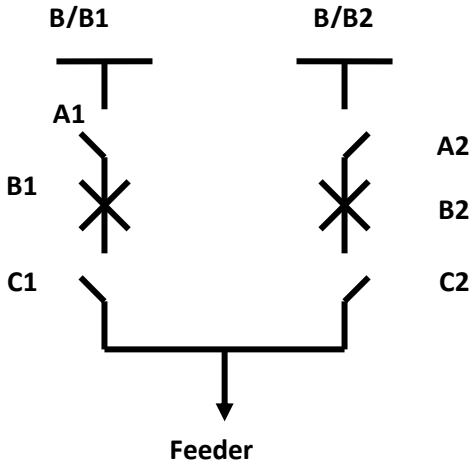


Fig.2 System II

III. Double Bus Bar with Breaker and a Half Scheme (System III)

This system is a compromise between the two previous systems. It proves to be the most reliable as well as economical arrangement of feeding two feeders in a ring system. Fig.3 shows the layout of this system (system III). The reliability of the feeders can take the following forms:

$$P_{III} = P\,[(A1 \cap B_1 \cap C_1)\ U\ (A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3)]$$

$$= P\,(A1 \cap B_1 \cap C_1) + P\,(A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3)$$
$$- P\,(A1 \cap B_1 \cap C_1)\ P\,(A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3)$$

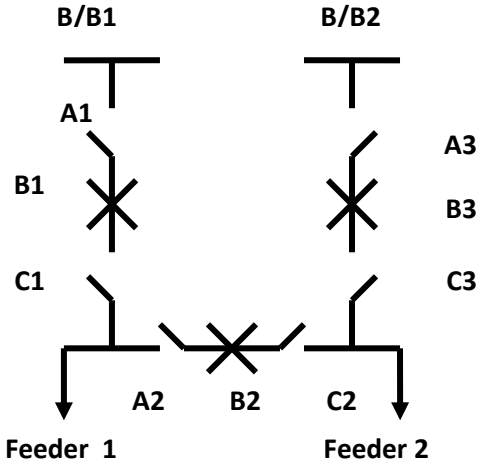


Fig.3 System III

$$= P_{a1}\,Pb_1\,Pc_1 + P_{a2}\,Pb_2\,Pc_2\,P_{a3}\,Pb_3\,Pc_3 - P_{a1}\,Pb_1\,Pc_1\,P_{a2}\,Pb_2\,Pc_2\,P_{a3}\,Pb_3\,Pc_3 \qquad (8)$$

The unavailability of the system is given by,

$$Q_{III} = 1 - P_{III} = 1 - P_{a1}\,Pb_1\,Pc_1 - P_{a2}\,Pb_2\,Pc_2\,P_{a3}\,Pb_3\,Pc_3$$
$$+ P_{a1}\,Pb_1\,Pc_1\,P_{a2}\,Pb_2\,Pc_2\,P_{a3}\,Pb_3\,Pc_3$$

If $P_{a1} = P_{a2} = P_{a3} = Pc_1 = Pc_2 = Pc_3 = P_a$

$Pb_1 = Pb_2 = Pb_3 = Pb$ thus, and

$$P_{III} = P_a^{\,2}\,Pb + P_a^{\,4}\,Pb^2 - P_a^{\,6}\,Pb^3$$

$$= P_I + P_I^2 + P_I^3 \quad (9)$$

The unavailability of the system is given by

$$Q_{III} = 1 - P_{III} = (1 - P_I)(1 - P_I^2) \quad (10)$$

The yearly outage time is given by

$$T_{fIII} = 8760 (1 - P_{III})$$

The fol1owing Table -1 shows the reliability and failure time/ year of different values of elements reliability.

Table -1 System parameters for breaker and a half scheme

$P_a = P_b = P_c$	P_{III}	T_{fIII} (hours)
0.9000	0.873020	1112.342
0.9900	0.998261	15.22759
0.9990	0.999982	0. 158729
0.9999	0.999999	0.00876

The ring bus bar scheme has the appearance of possessing almost all the desirable features. As the ring is closed each, feeder has two sources of supply, and any circuit breaker may be taken out of service without affecting the supply. If, however a second breaker opens, a number of circuits will be lost, and for this reason the ring scheme should be limited to four or six circuits for reliable distribution.

Consider the scheme shown in Fig.4, The availability of Feeder 1 can be found as follows:

$$P_{F1} = P [(A1 \cap B_1 \cap C_1) \cup (A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})]$$

$$= P (A1 \cap B_1 \cap C_1) + P (A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})$$

$$- P (A1 \cap B_1 \cap C_1 \cap A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})$$

$= P_{a1}\, Pb_1\, Pc_1 + P_{a2}\, Pb_2\, Pc_2 + P_{a3}\, Pb_3\, Pc_3 + \ldots\ldots\ldots + Pa_{(n+1)}\, P_{b(n+1)}\, Pc_{(n+1)}$

$- P_{a1}\, Pb_1\, Pc_1\, P_{a2}\, Pb_2\, Pc_2\, P_{a3}\, Pb_3\, Pc_3 \ldots\ldots\ldots\ldots P_{a(n+1)}\, P_{b(n+1)}\, Pc_{(n+1)}$

If $P_{a1} = P_{a2} = P_{a3} = \ldots\ldots = P_{a(n+1)} = Pa$, $Pb_1 = Pb_1 = Pb_1 = \ldots\ldots P_{b(n+1)} = P_b$ and $Pc_1 = Pc_2 = Pc_3 = \ldots\ldots Pc_{(n+1)} = Pc$, keeping in mind that $P_I = P_a^2\, Pb$

therefore, $P_{F1} = P_I + P_I^{n} - P_I^{(n+1)}$ (12)

and , $Q_{F1} = 1 - P_{F1}$ (13)

Also the availability of feeder 2 is given by:

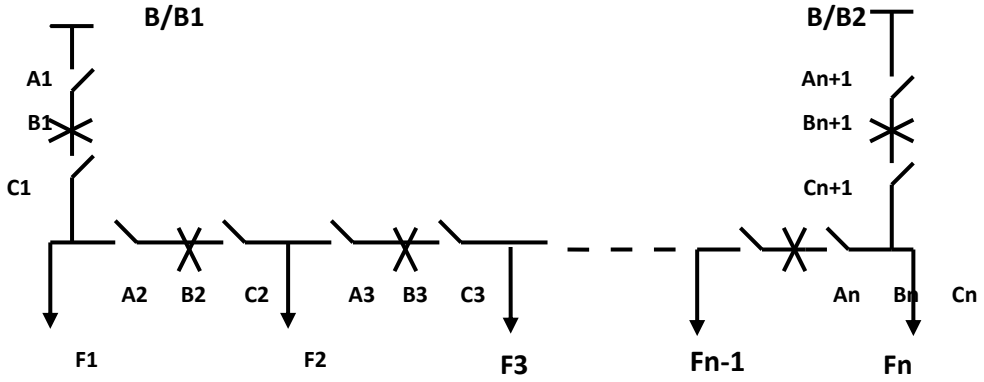


Fig.4 Ring-bus system with n outgoing power feeders.

$P_{F2} = P\,[(A1 \cap B_1 \cap C_1) \cap (A_2 \cap B_2 \cap C_2)\ U\ (A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})]$

$= P\,(A1 \cap B_1 \cap C_1 \cap A_2 \cap B_2 \cap C_2) + P(A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})$

$- P\,(A1 \cap B_1 \cap C_1 \cap A_2 \cap B_2 \cap C_2 \cap A_3 \cap B_3 \cap C_3 \cap \ldots\ldots\ldots \cap A_{n+1} \cap B_{n+1} \cap C_{n+1})$

$= P_{a1}\, Pb_1\, Pc_1\, P_{a2}\, Pb_2\, Pc_2 + P_{a3}\, Pb_3\, Pc_3 \ldots\ldots\ldots P_{a(n+1)}\, P_{b(n+1)}\, Pc_{(n+1)}$

$- P_{a1}\, Pb_1\, Pc_1\, P_{a2}\, Pb_2\, Pc_2\, P_{a3}\, Pb_3\, Pc_3 \ldots\ldots\ldots\ldots P_{a(n+1)}\, P_{b(n+1)}\, Pc_{(n+1)}$

Similarly, $P_{F2} = P_I^2 + P_I^{(n-1)} - P_I^{(n+1)}$ and , $Q_{F2} = 1 - P_{F2}$

Now, for any feeder k in the ring system, the availability can be found as,

$$P_{Fk} = P\,[(A1\cap B_1\cap C_1\cap A_2\cap B_2\cap C_2 \cap \ldots\ldots\ldots\cap A_k\cap B_k\cap C_k)\ U(A_{k+1}\cap B_{k+1}\cap C_{k+1} \cap \ldots\ldots\ldots\cap A_{n+1}\cap B_{n+1}\cap C_{n+1})]$$

$$= P\,(A1\cap B_1\cap C_1\cap A_2\cap B_2\cap C_2\cap \ldots A_k\cap B_k\cap C_k) + P\,(A_{k+1}\cap B_{k+1}\cap C_{k+1}\cap \ldots\ldots\ldots\cap A_{n+1}\cap B_{n+1}\cap C_{n+1})$$

$$- P\,(A1\cap B_1\cap C_1\cap A_2\cap B_2\cap C_2\cap \ldots\ldots A_k\cap B_k\cap C_k\cap \ldots\ldots\ldots\cap A_{n+1}\cap B_{n+1}\cap C_{n+1})$$

$$= P_{a1}\,Pb_1\,Pc_1\,P_{a2}\,Pb_2\,Pc_2 \ldots P_{ak}\,Pb_k\,Pc_k + P_{ak+1}\,Pb_{k+1}\,Pc_{k+1} \ldots\ldots\ldots P_{a(n+1)}\,P_{b(n+1)}\,Pc_{(n+1)} - P_{a1}\,Pb_1\,Pc_1\,P_{a2}\,Pb_2\,Pc_2 \ldots P_{ak}\,Pb_k\,Pc_k \ldots\ldots\ldots\ldots\ldots\ldots P_{a(n+1)}\,P_{b(n+1)} \ldots\ldots\ldots P_{c(n+1)}$$

Using the same assumptions given in eq. (9) above we get,

$$P_{Fk} = P_l^{\,k} + P_l^{\,(n+1-k)} - P_l^{\,(n+1)} \qquad (14)$$

and , $Q_{Fk} = 1 - P_{Fk}$ (15)

In general, the reliability of the jth feeder in a ring-bus system can be obtained from the following general formula:

$$P_j = P_l^{\,j} + P_l^{\,(n+1-j)} - P_l^{\,(n+1)} \qquad (16)$$

The unavailability is given by,

$$Q_j = 1 - P_j = 1 - P_l^{\,j} - P_l^{\,(n+1-j)} + P_l^{\,(n+1)}$$

$$= (1 - P_l^{\,j})\,(1 - P_l^{\,(n+1-j)})$$

the yearly expected outage time in hours for the jth feeder in the ring –bus system is

$$T_{fj} = 8760\,Q_j = 8760\,(1 - P_l^{\,j})\,(1 - P_l^{\,(n+1-j)}) \qquad (17)$$

These equations are programmed and worked on Matlab program in order to study the reliability characteristics for each feeder in the ring system when the number of the feeders is varied. The results are given in Tables -2 and 3.

Table -2 Variation of the individual feeder availability of a 3-feeder ring system with the element reliability

Element Reliability	Feeder No.1		Feeder No.2		Feeder No.3	
	PF1	TF1	PF2	TF2	PF3	TF3
0.9000	0.83399	1454.241	0.78045	1923.23	0.83390	1454.241
0.9900	0.997431	22.503	0.99657	30.000	0.997431	22.503
0.9990	0.999973	0.237050	0.99996	0.314326	0.999973	0.237070
0.9999	0.999999	0.002088	0.99999	0.004177	0.9999999	0.002088

Table -3 Variation of feeder's outage time for a 7-feeder ring system with element reliability =0.9900

Outage Time in Hours / year= $8760(1-P_F)$							
n	Feeder-1	Feeder-2	Feeder-3	Feeder-4	Feeder-5	Feeder-6	Feeder-7
1	7.72866	…….	……	……	……	…….	……
2	15.227	15.227	……	…….	……	……..	……
3	22.503	30.000	22.503	……	……	……	…….
4	29.562	44.336	44.336	29.562	……	…….	……
5	36.413	58.245	65.521	58.245	36.413	……..	……
6	43.057	71.74	86.0773	86.0773	71.74	43.057	……..
7	49.507	84.837	106.024	113.081	106.024	84.837	49.507

The effect of variation of the number of feeders in the system on the reliabi1ities on The individual feeders is shown in Fig.5 (system with 9 feeders) . It is noticed that the reliability of each feeder is decreased as the number of the outgoing feeders is increased .Also it can be seen that the most reliable feeders are those which are near the feeding points, feeders 1 and 9 in this

case.Fig.6 shows the Variation of yearly expected feeder outage time for different values of n. However from redundancy point of view the ring system is consider as a low level redundancy.

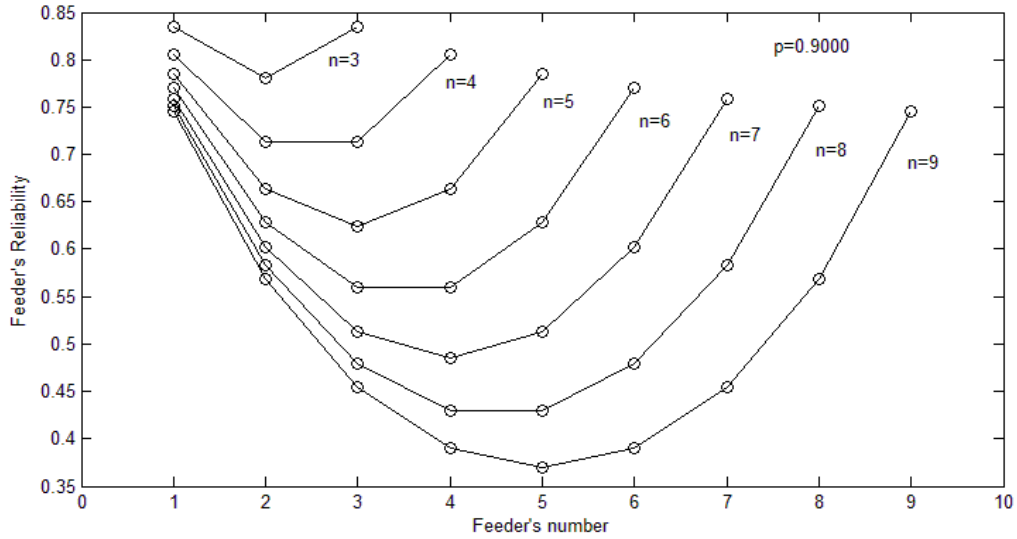


Fig. 5 Reliability variation of individual feeder with the number of feeders in a ring-bus system.

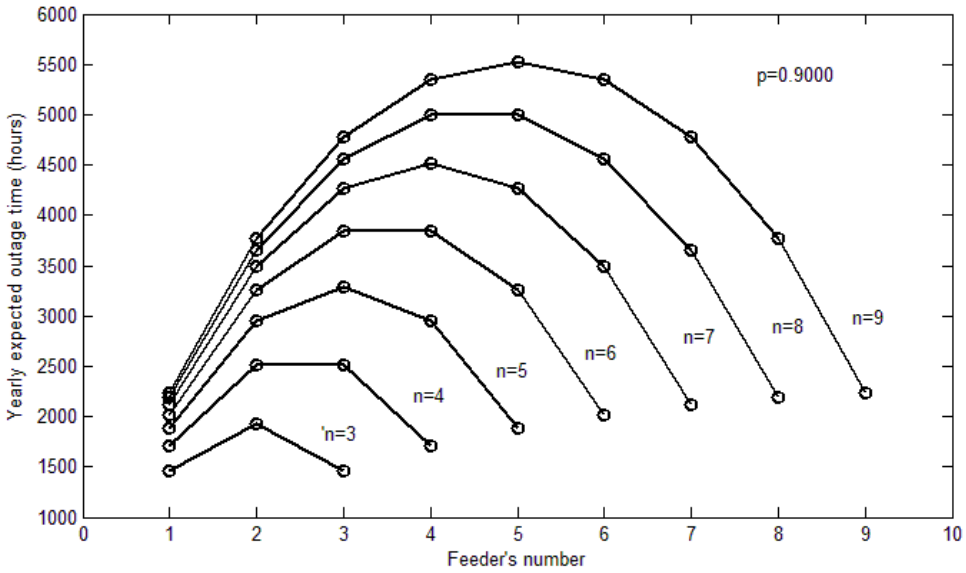


Fig.6 Variation of yearly expected feeder outage time for different values of n.

4. Practical Applications of the General Ring-Bus Formulae

The general ring-bus reliability formulae given in eqs.(14-17) are now applied to different practical power supply schemes in order to find the reliability, unreliability and outage time of the individual system feeders. The application of these formulae for these schemes is done by replacing (n) in eq.(13) by its actual number which represents the number of feeders supplying loads in the scheme; this may be done as follows:

(1) For the double bus bar double breaker scheme shown in Fig.2 , the general ring-bus reliability formula can be applied directly by setting n =1(since the number of feeders =1) and j= 1. The reliability of this scheme which may be designated by R_{II} is therefore ,

$$R_{II} = 2\,R_I - R_I^2$$

The unreliability of the system is given by

$$Q_{II} = 1 - R_{II} = 1 - 2\,R_I + R_I^2$$

and the yearly expected outage time in hours is given by

$$T_{fII} = 8760\,Q_{II} = 8760\,(1 - R_I)^2$$

(2) For the double bus bar with breaker and a half scheme shown in Fig.3, the general ring bus formula can be applied by setting n=2 (since the number of feeders =2) and j=1, 2. The reliability of feeder 1 and 2 in this scheme are therefore ,

$$R_1 = R_2 = R_I + R_I^2 - R_I^3$$

And the reliability of this scheme which may be designated by R_{III} is therefore,

$$R_{III} = R_I + R_I^2 - R_I^3$$

The unreliability of the system is given by

$$Q_{III} = 1 - R_{III} = 1 - R_I - R_I^2 + R_I^3$$

and the yearly expected outage time in hours is given by

$$T_{fIII} = 8760\,Q_{III} = 8760\,(1 - R_I - R_I^2 + R_I^3)$$

11

Reliability Evaluation of Interconnected Substations

Consider two substations connected by two lines as shown in Fig-1. Substation 1 has three breakers protecting two feeders (circuits) and as a result, is referred to as a "breaker and a half scheme". From a reliability point of view, let's consider that this system "Works" if power is delivered from at least one of the buses (BB1 or BB2).

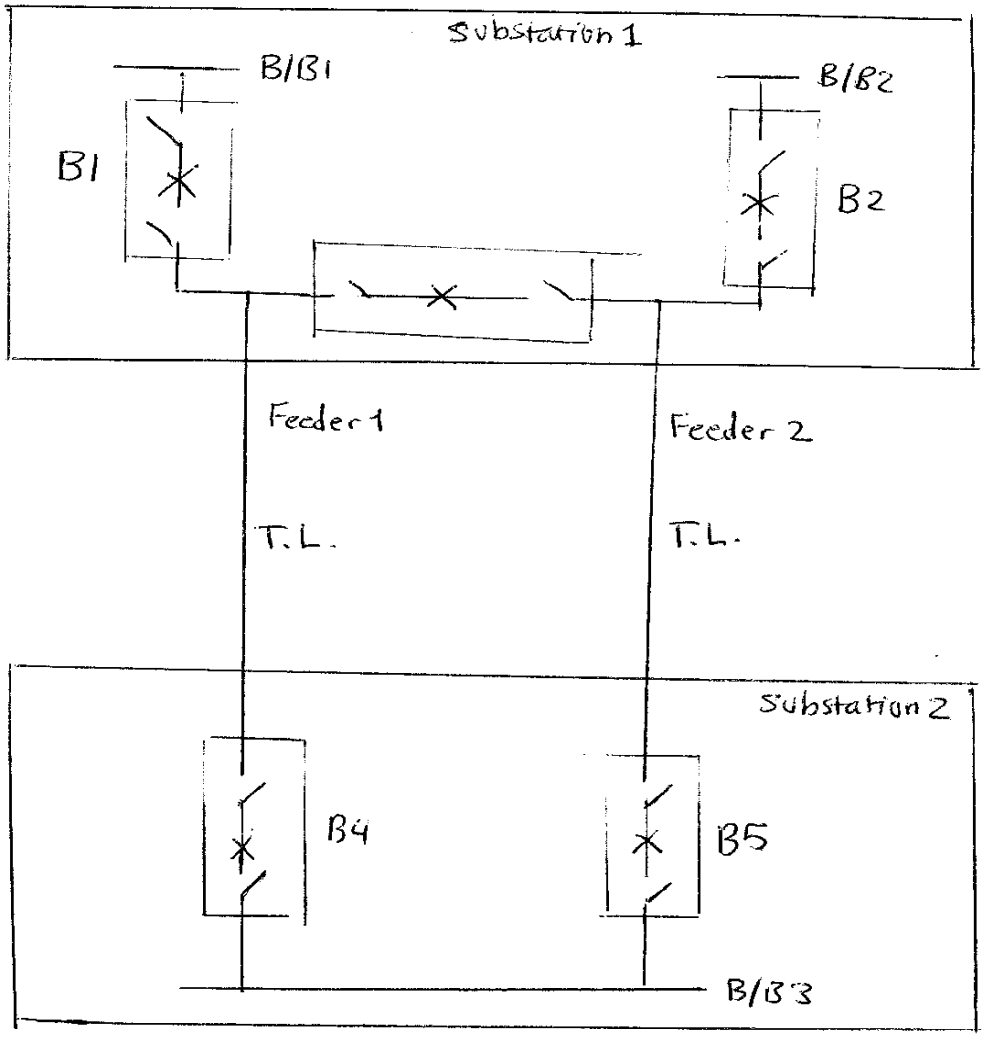


Fig-1 Substation Configuration

Also, the scheme "fails" if power cannot be delivered from either bus.

We want to construct the logic diagram for this system. One observes that the following constitue failure modes for this system:

(B_1, B_2), (B_4, B_5), (B_4, B_2, B_3), (B_5, B_1, B_3).

We may represent these failure modes using the following logic diagram

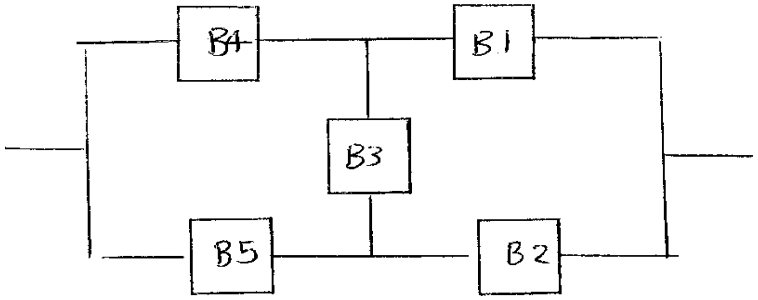


Fig.2: substation logic diagram

This is a bridge diagram which can be solved by one of the following techniques:

1. Set theory (Probabilistic approach)
2. Cut Set method
3. Tie-Set method
4. Decomposition
5. Delta-Star transformation
6. Event trees
7. Fault trees
8. Connection matrix method

1. Set theory solution to bridge circuit :

Path 1	through	$(B4 \cap B1)$
path 2	through	$(B5 \cap B2)$
path 3	through	$(B4 \cap B3 \cap B2)$
path 4	through	$(B5 \cap B3 \cap B1)$

The network is available if one or more of the above paths are available, and this is given by :

$$(B4 \cap B1) \cup (B5 \cap B2) \cup (B4 \cap B3 \cap B2) \cup (B5 \cap B3 \cap B1)$$

The probability of this condition is

$$\begin{aligned} P = {} & P(B4 \cap B1) + P(B5 \cap B2) + P(B4 \cap B3 \cap B2) + P(B5 \cap B3 \cap B1) \\ & - P(B4 \cap B1 \cap B5 \cap B2) - P(B4 \cap B1 \cap B4 \cap B3 \cap B2) \\ & - P(B4 \cap B1 \cap B5 \cap B3 \cap B1) - P(B5 \cap B2 \cap B4 \cap B3 \cap B2) \\ & - P(B5 \cap B2 \cap B5 \cap B3 \cap B1) - P(B4 \cap B3 \cap B2 \cap B5 \cap B3 \cap B1) \\ & + P(B4 \cap B1 \cap B5 \cap B2 \cap B4 \cap B3 \cap B2) \\ & + P(B5 \cap B2 \cap B4 \cap B3 \cap B2 \cap B5 \cap B3 \cap B1) \\ & + P(B4 \cap B5 \cap B2 \cap B5 \cap B3 \cap B1 \cap B4 \cap B1) \\ & + P(B5 \cap B3 \cap B1 \cap B4 \cap B1 \cap B5 \cap B2) \\ & - P(B4 \cap B1 \cap B5 \cap B2 \cap B4 \cap B3 \cap B2 \cap B5 \cap B3 \cap B1) \end{aligned}$$

$$\begin{aligned} = {} & P_{B4} \cdot P_{B1} + P_{B5} P_{B2} + P_{B4} P_{B3} P_{B2} + P_{B5} P_{B3} P_{B1} \\ & - P_{B1} P_{B2} P_{B4} P_{B5} - P_{B1} P_{B2} P_{B3} P_{B4} - P_{B1} P_{B3} P_{B4} P_{B5} \\ & - P_{B2} P_{B3} P_{B4} P_{B5} - P_{B1} P_{B2} P_{B3} P_{5} - P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} \\ & + P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} + P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} \\ & + P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} + P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} \\ & - P_{B1} P_{B2} P_{B3} P_{B4} P_{B5} \end{aligned}$$

6

2. Cut Set Method

Definitions:

- A cut set K is a set of components whose failure results in system failure. i.e removal of all components in any cutset disconnects the input from the output in the logical diagram.
- A minimal cut set C is a cutset where the set remaining after a removal of any its elements is no longer a cut set. This means that all components of a minimal cut set must be failed to cause system failure.

Example: Find all cutsets of Figure below:

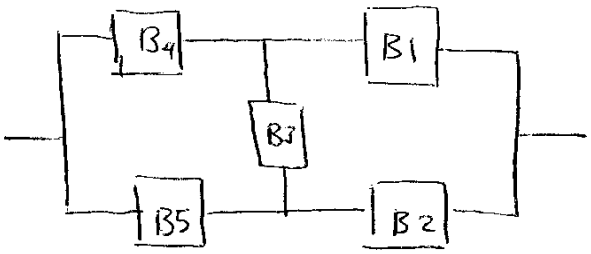


Fig.1

$(B4, B5)$, $(B1, B2)$, $(B4, B3, B2)$, $(B1, B3, B5)$

$$Q_{syst} = P(F_{sys}) = P(F_{C1} \cup F_{C2} \cup F_{C3} \cup F_{C4})$$

- We can develop a logical model for our system as:

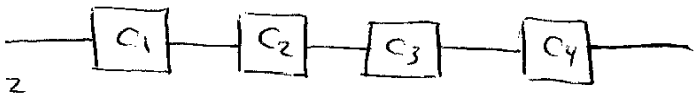
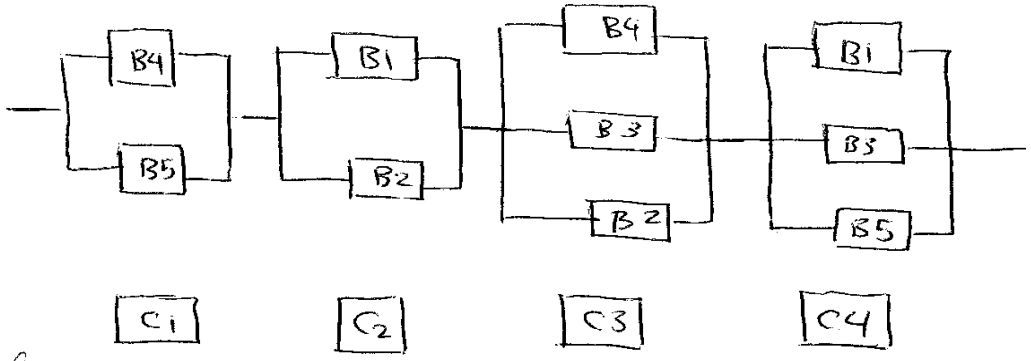


Fig.2

Equivalent cutset logic diagram

- All cutset blocks can be modeled logically as parallel combination of their constituent components So we can expand the logic diagram of Fig 2 as:



we have four events :

$$Q_{sys} = P(C_1 \cup C_2 \cup C_3 \cup C_4)$$

$$= P(C_1) + P(C_2) + P(C_3) + P(C_4)$$
$$- P(C_1 \cap C_2) - P(C_1 \cap C_3) - P(C_1 \cap C_4)$$
$$- P(C_2 \cap C_3) - P(C_2 \cap C_4) - P(C_3 \cap C_4)$$
$$+ P(C_1 \cap C_2 \cap C_3) + P(C_1 \cap C_2 \cap C_4) + P(C_1 \cap C_3 \cap C_4)$$
$$+ P(C_2 \cap C_3 \cap C_4)$$
$$- P(C_1 \cap C_2 \cap C_3 \cap C_4)$$

The reliability of the system :

$$R_{sys} = R_{C_1} R_{C_2} R_{C_3} R_{C_4}$$

$$R_{C_1} = 1 - Q_{C_1} = 1 - (1 - P_{B4})(1 - P_{B5}) = 1 - q_{B4} q_{B5}$$

$$R_{C_2} = 1 - Q_{C_2} = 1 - (1 - P_{B1})(1 - P_{B2}) = 1 - q_{B1} q_{B2}$$

$$R_{C_3} = 1 - Q_{C_3} = 1 - (1 - P_{B4})(1 - P_{B3})(1 - P_{B2}) = 1 - q_{B4} q_{B3} q_{B2}$$

$$R_{C_4} = 1 - Q_{C_4} = 1 - (1 - P_{B1})(1 - P_{B3})(1 - P_{B5}) = 1 - q_{B1} q_{B3} q_{B5}$$

Note that :

$$Q_{C_1} = P(C_1) = q_{B4} q_{B5}$$
$$Q_{C_2} = P(C_2) = q_{B1} q_{B2}$$
$$Q_{C_3} = P(C_3) = q_{B4} q_{B3} q_{B2}$$
$$Q_{C_4} = P(C_4) = q_{B1} q_{B3} q_{B5}$$

8

Example : Consider the 4-bus system shown in Figure below, use the cut set method to determine the reliability and, unreliability of the system. Assume that the reliability of the generator $R_G = 0.92$ and the reliability of each line $R_{L1} = R_{L2} = R_{L3} = R_{L4} = R_{L5} = 0.95$.

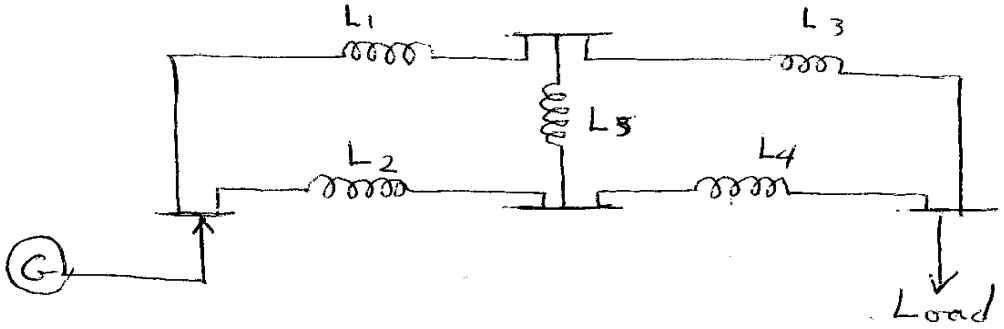


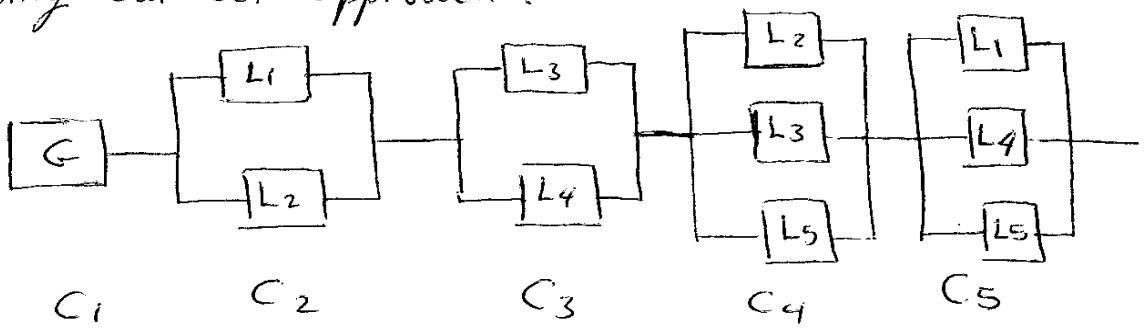
Fig. 4-Bus System

Solution :

$R_G = 0.92 \Rightarrow Q_G = 1 - R_G = 1 - 0.92 = 0.08$

$R_L = 0.95 \Rightarrow Q_L = 1 - R_L = 1 - 0.95 = 0.05$

Using cut set approach :



$$Q_{syst} = P(C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5)$$

$$= P(C_1) + P(C_2) + P(C_3) + P(C_4) + P(C_5)$$

$$- [P(C_1 \cap C_2) + P(C_1 \cap C_3) + P(C_1 \cap C_4) + P(C_1 \cap C_5) + P(C_2 \cap C_3) + P(C_2 \cap C_4) + P(C_2 \cap C_5) + P(C_3 \cap C_4) + P(C_3 \cap C_5) + P(C_4 \cap C_5)]$$

$$+ [P(C_1 \cap C_2 \cap C_3) + P(C_1 \cap C_2 \cap C_4) + P(C_1 \cap C_2 \cap C_5) + P(C_2 \cap C_3 \cap C_4) + P(C_2 \cap C_3 \cap C_5) + P(C_3 \cap C_4 \cap C_5) + P(C_1 \cap C_3 \cap C_4) + P(C_1 \cap C_4 \cap C_5) + P(C_2 \cap C_4 \cap C_3) + P(C_1 \cap C_3 \cap C_5)]$$

9

$$- \left[P(C_1 \cap C_2 \cap C_3 \cap C_4) + P(C_1 \cap C_2 \cap C_3 \cap C_5) + P(C_2 \cap C_3 \cap C_4 \cap C_5) + P(C_3 \cap C_4 \cap C_5 \cap C_1) + P(C_1 \cap C_2 \cap C_4 \cap C_5) \right] + P(C_1 \cap C_2 \cap C_3 \cap C_4 \cap C_5)$$

But $P(C_1) = Q_{C_1} = Q_G = 0.08$

$P(C_2) = Q_{C_2} = Q_{L_1} \cdot Q_{L_2} = (0.05)^2$

$P(C_3) = Q_{C_3} = Q_{L_3} \cdot Q_{L_4} = (0.05)^2$

$P(C_4) = Q_{C_4} = Q_{L_2} Q_{L_3} Q_{L_5} = (0.05)(0.05)(0.05) = (0.05)^3$

$P(C_5) = Q_{C_5} = Q_{L_1} Q_{L_4} Q_{L_5} = (0.05)(0.05)(0.05) = 0.05)^3$

$$\therefore Q_{syst} = 0.08 + (0.05)^2 + (0.05)^2 + (0.05)^3 + (0.05)^3$$

$$- \left[0.08 \times (0.05)^2 + 0.08 (0.05)^2 + 0.08 \times (0.05)^3 + 0.08 \times (0.05)^3 + (0.05)^4 + (0.05)^4 + (0.05)^4 + (0.05)^4 + (0.05)^4 + (0.05)^5 \right]$$

$$+ \left[0.08 \times (0.05)^4 + 0.08 (0.05)^4 + 0.08 (0.05)^4 + (0.05)^5 + (0.05)^5 + (0.05)^5 + 0.08 \times (0.05)^4 + 0.08 \times (0.05)^5 + (0.05)^3 + 0.08 \times (0.05)^4 \right]$$

$$- \left[0.08 \times (0.05)^5 + 0.08 \times (0.05)^5 + (0.05)^5 + 0.08 \times (0.05)^5 + 0.08 \times (0.05)^5 \right]$$

$$+ \left[0.08 \times (0.05)^5 \right]$$

$$\Rightarrow Q_{syst} = 0.08480825$$

$$P_{syst} = 1 - Q_{syst} = 0.91519175$$

Expected Annual outage time:

$$T_{syst} = Q_{sys} \times 8760 = 8760 \times 0.08480825$$

$$= 742.86 \text{ hours}$$

10

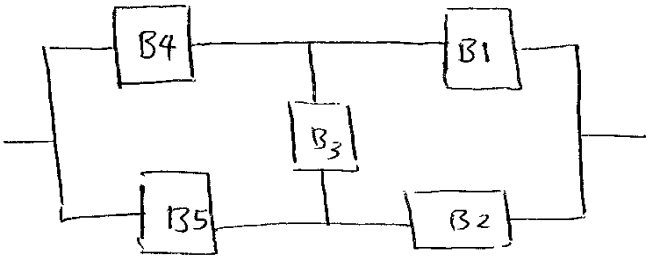
12

3. Tie set Method

Definitions

- A tie set V is a set of components whose success results in system success. The presence of all components in any tieset "connects" the input to the "output" in the logical diagram.
- A minimal tieset T is a tieset where the set remaining after a removal of any of its elements is no longer a tieset. This means that all components of a minimal tieset must succeed to cause system success.

Example : Find all tiesets for figure below:

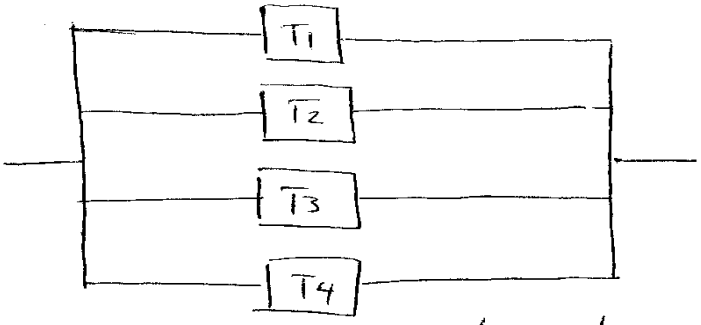


Solution: The tie sets are:

$$\underset{T_1}{(B_4, B_1)}, \underset{T_2}{(B_5, B_2)}, \underset{T_3}{(B_4, B_3, B_2)}, \underset{T_4}{(B_1, B_3, B_5)}.$$

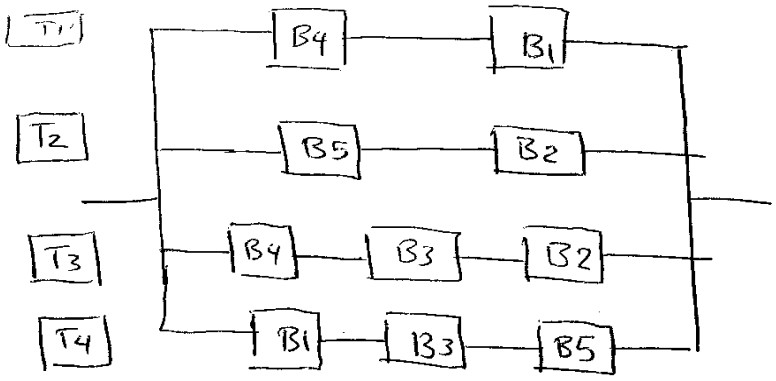
$$R_{sys} = P(S) = P(T_1 \cup T_2 \cup T_3 \cup T_4)$$

We can develope a logical model for the above system:



Equivalent Tie set logic diagram

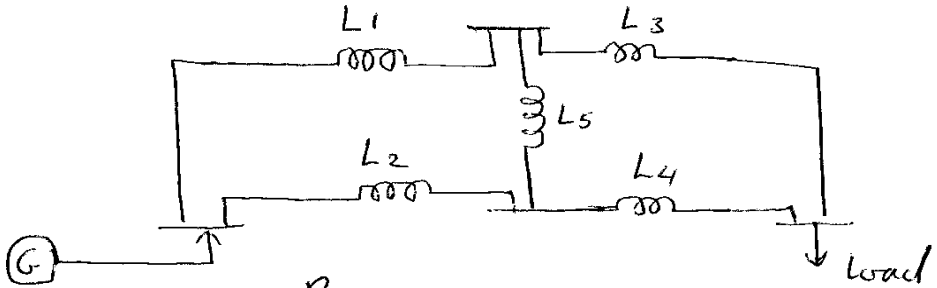
All tieset blocks can be modeled logically as a series combination of their components. So we can expand the logic diagram as follows:



$$P_{sys} = P(T_1 \cup T_2 \cup T_3 \cup T_4)$$

$$= P(T_1) + P(T_2) + P(T_3) + P(T_4)$$
$$- [P(T_1 \cap T_2) + P(T_1 \cap T_3) + P(T_1 \cap T_4) + P(T_2 \cap T_3) + P(T_2 \cap T_4) + P(T_3 \cap T_4)]$$
$$+ [P(T_1 \cap T_2 \cap T_3) + P(T_1 \cap T_2 \cap T_4) + P(T_2 \cap T_3 \cap T_4) + P(T_1 \cap T_3 \cap T_4)]$$
$$- P(T_1 \cap T_2 \cap T_3 \cap T_4) .$$

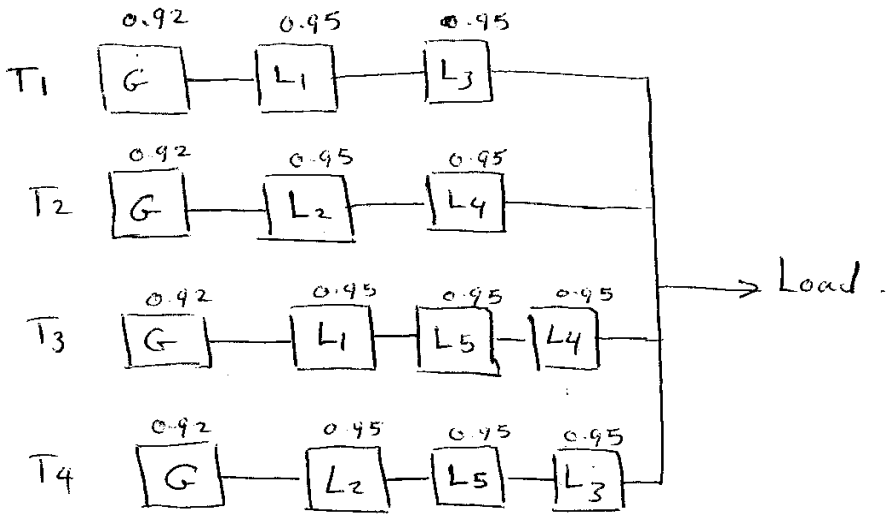
Numerical Example: For the 4-bus system shown find the system reliability, unreliability and the expected annual outage time in hours.



$R_L = 0.95$

$R_G = 0.92$

Solution : Using tiesets approach :



$$P_{syst} = P(T_1 \cup T_2 \cup T_3 \cup T_4)$$

$$= P(T_1) + P(T_2) + P(T_3) + P(T_4)$$

$$- [P(T_1 \cap T_2) + P(T_1 \cap T_3) + P(T_1 \cap T_4) + P(T_2 \cap T_3) + P(T_2 \cap T_4) + P(T_3 \cap T_4)]$$

$$+ [P(T_1 \cap T_2 \cap T_3) + P(T_1 \cap T_2 \cap T_4) + P(T_2 \cap T_3 \cap T_4) + P(T_1 \cap T_3 \cap T_4)]$$

$$- [P(T_1 \cap T_2 \cap T_3 \cap T_4)]$$

$$\Rightarrow P_{syst} = 0.92 \times (0.95)^2 + 0.92 \times (0.95)^2 + 0.92 \times (0.95)^3 + 0.92 \times (0.95)^3 - [0.92 \times (0.95)^4 + 0.92 \times (0.95)^4 + 0.92 \times (0.95)^4 + 0.92 \times (0.95)^4 + 0.92 \times (0.95)^4 + 0.92 (0.95)(0.95)^5] + [0.92 \times (0.95)^5 + 0.92 \times (0.95)^5 + 0.92 \times (0.95)^5 + 0.92 \times (0.95)^5] - [0.92 \times (0.95)^5]$$

$$= 0.915198175$$

$$Q_{sys} = 1 - P_{sys} = 0.084801$$

$$T_{f\,sys} = 8760\, Q_{sys} = 742.86 \text{ hours}$$

-3-

Power System Reliability Dr. Mohammed Tawfeeq

Distribution Systems

4. The Fault Tree Approach:

Using the symbols of Fig.1, a fault tree analysis (FTA) evaluates undesired event (top event).

Fault trees are effectively used to provide a systematic way to view logical sequences of events that can lead to failure of complex system.

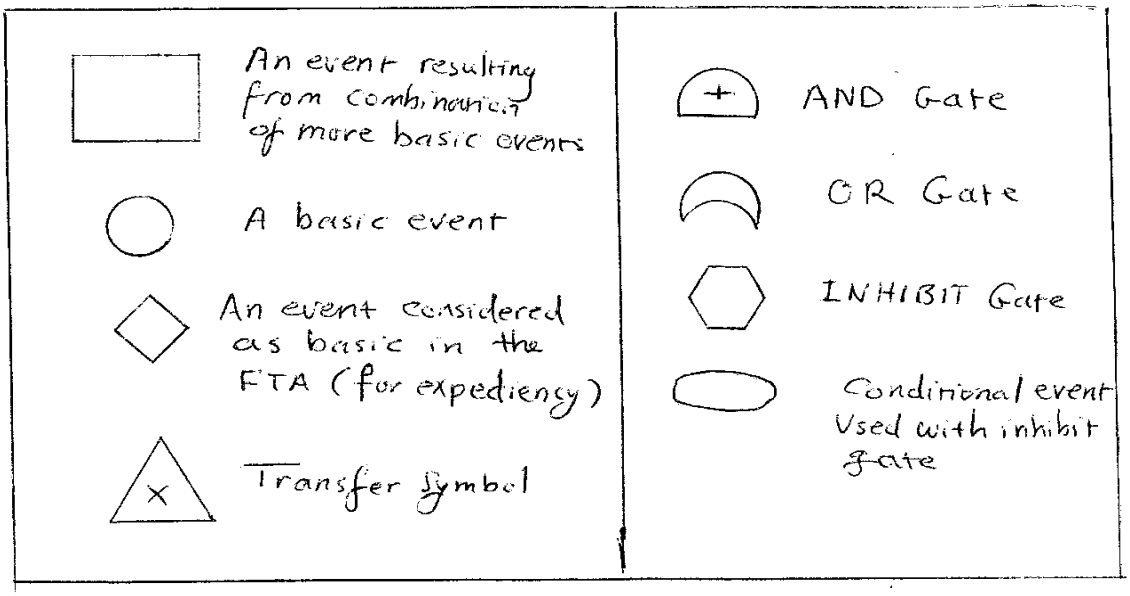

An event resulting from combination of more basic events
A basic event
An event considered as basic in the FTA (for expediency)
Transfer Symbol
AND Gate
OR Gate
INHIBIT Gate
Conditional event Used with inhibit gate

Fig.1 The Fault tree Gate and event Symbols.

Characteristics of fault trees:

1. It is a "top-down" analysis, where the top-event is specified first.
2. Does not necessarily contain all possible failure mode of the component of the system. Only those failure modes which contribute to the existance occurance of the top event are modeled.
3. A fault tree is an expression for Boolean logic. i.e, all basic events are binary, that is, either true or false.

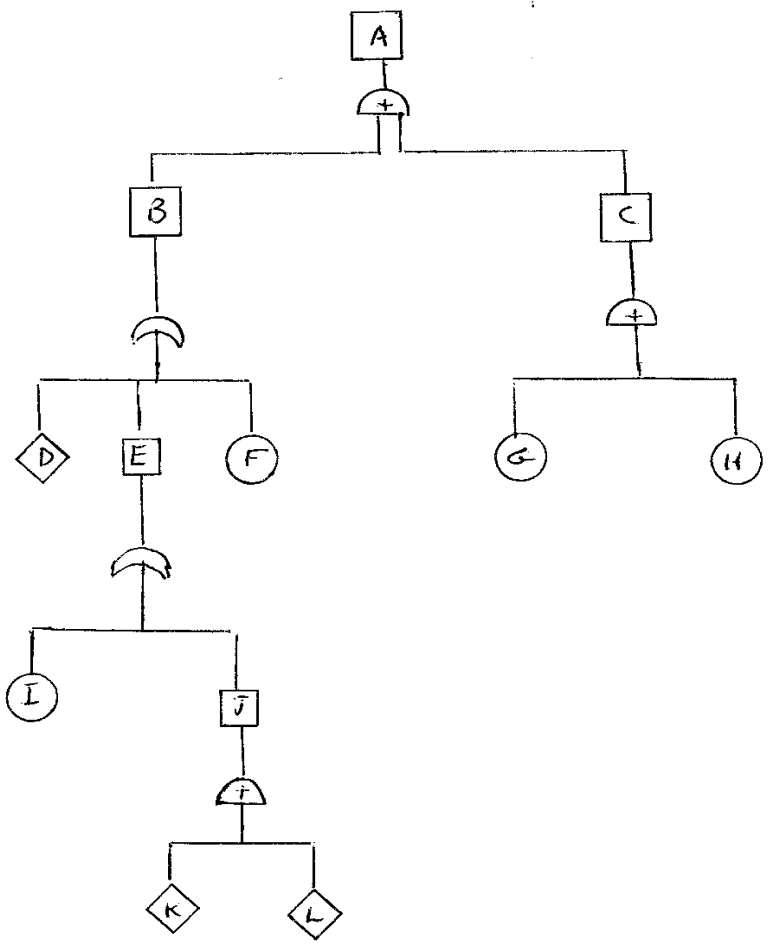
-1-

Example1: Determine the probability of event A in the fault tree of Fig.2. if all the basic events and events assumed to be basic are statistically independent and mutually exclusive of one another. The probabilities of the basic events are estimated to be as follows:

$P(D) = 0.05$ $P(H) = 0.12$

$P(F) = 0.15$ $P(I) = 0.10$ $P(L) = 0.05$.

$P(G) = 0.20$ $P(K) = 0.05$



Solution: $P(\bar{J}) = P(K) \cap P(L) = P(K) \times P(L) = (0.05) \times 0.02 = 0.0025$

$P(\bar{E}) = P(I) \cup P(\bar{J}) = P(I) + P(\bar{J}) = 0.10 + 0.0025 = 0.1025$

$P(B) = P(D) \cup P(\bar{E}) \cup P(F) = P(D) + P(\bar{E}) + P(F) = 0.05 + 0.1025 + 0.15 = 0.3025$

$P(C) = P(G) \cap P(H) = P(G) \times P(H) = (0.20)(0.12) = 0.0240$

$P(A) = P(B) \cap P(C) = P(B) \times P(C) = 0.3025 \times 0.0240 = 0.00726$

-- 2 --

Example 2 : Consider the power system shown in Fig.A. The generator at Station A represents power inflow that can be considered as perfectly reliable for purpose of this example. Define "system failure" to be:

- Station B is isolated or
- Station C is isolated or
- The combined load of station B and C are carried by single circuit

Draw a fault tree for this system.

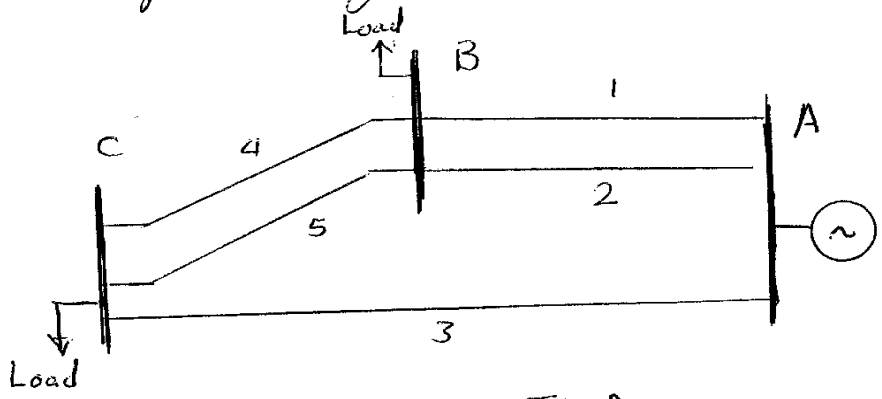
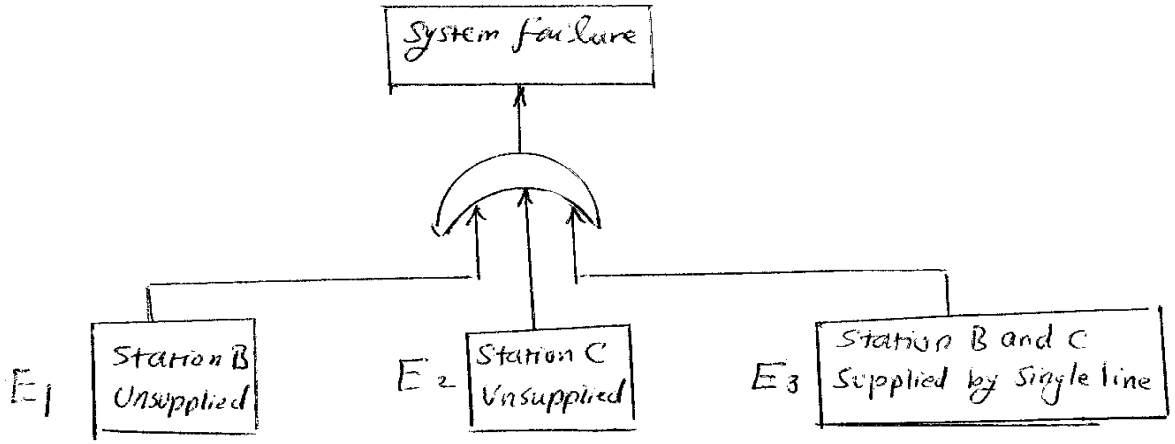


Fig.A

Solution : The top event is "system failure". Based on the given definition, we may immediately draw the first level of a tree as follows:



First Level.

Now one proceeds by analyzing each of the three ways that system failure may occur.

1. For event E_1 to occur it requires three events:
   a. loss of circuit 1 (basic event).
   b. loss of circuit 2 (basic event).
   c. loss of supply from Station C (intermediate event)

   Because of this failure mode requires all of these three events to occur, we flow them through an AND gate.

2. Occurance of the event (E_2) requires two events:
   a. Loss of circuit 3 (basic event)
   b. Loss of supply from Station B (intermediate event)

   Because this failure mode requires both of these two events to occur, we flow them through an AND gate.

3. Occurance of the event E_3 in three different ways:
   a. Supply from circuit 1 only (intermediate event)
   b. Supply from circuit 2 only (intermediate event) or
   c. Supply from circuit 3 only (intermediate event)

   Because this failure mode occurs if any of these three events occur, we flow them through OR gate.

In this way we can proceed to draw the second (third level) and finally the full tree as shown in Fig. B below:

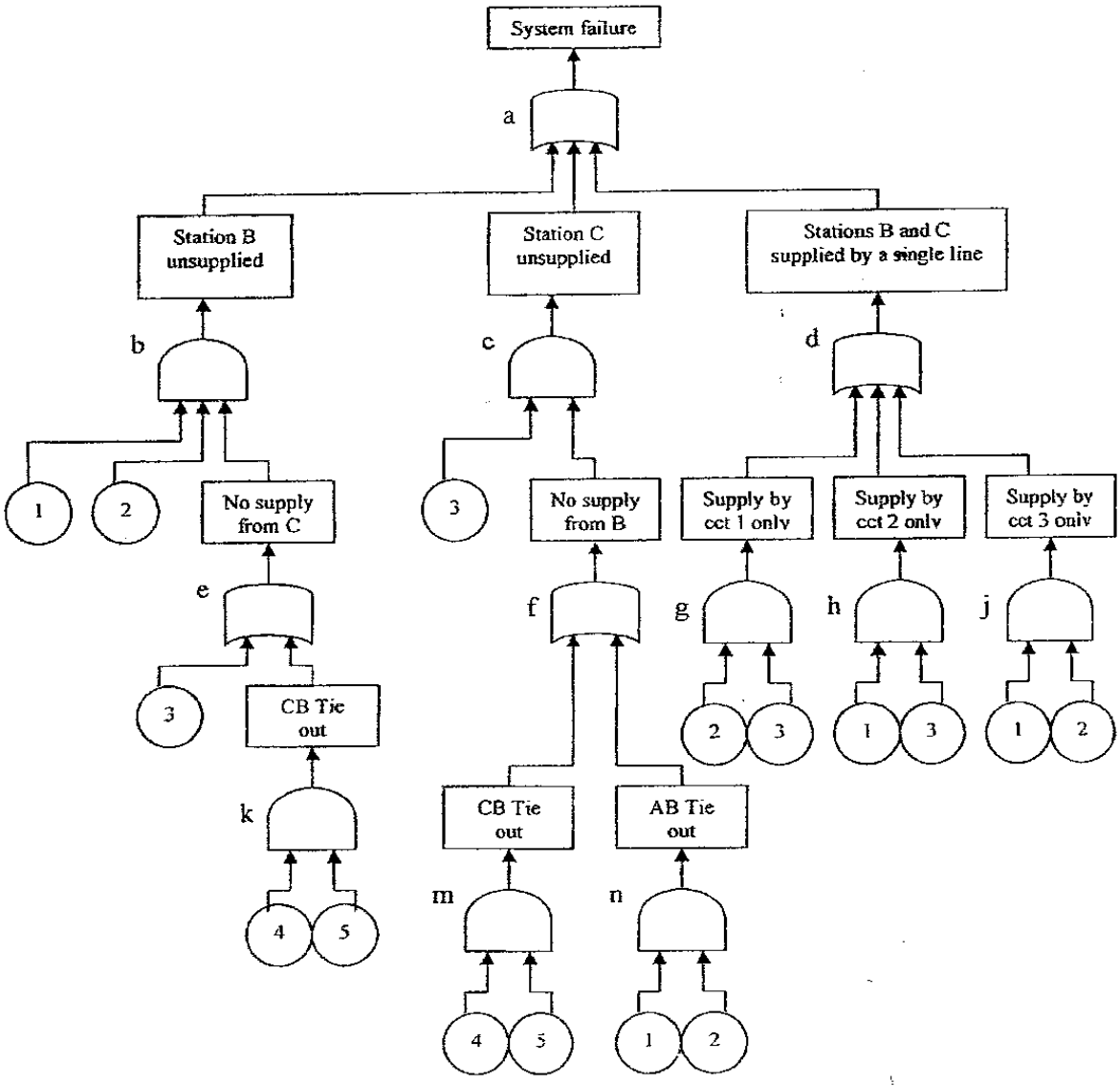


Fig. B Fault tree

13 Generation System Reliability Evaluation

Important Terms and Definitions

1. Unit : is a group of components which are functionally related. (Generating unit, steam turbine unit, ...).
2. Component : is a device which performs a major operating function. Some examples are, transformer, switchgear, circuit breaker, isolator, resistor, inductor, capacitor, transistor - etc.
   The components can be classified into two groups:
   (a) Non-reparable component
   (b) repairable component.
3. Units and Components State of events
   Unit or component can be in two state:
   (a) In-service State (UP State)
   (b) Outage State (DOWN State)
4. Outage types
   The outages or (not in service) states are:
   (a) Forced outages
      (i) Transient forced outage
      (ii) Permanent forced outage
      (iii) Temporary forced outage
   (b) Scheduled Outage
      A planned outage for purposes of testing, maintenance, construction or repair.

5. Availability and Forced Outage Rate of Single Repairable Unit - Markov Modeling

Consider a single generating Unit shown in Fig.1.

Let the set of service time is:

$S_s = \{T_{U1}, T_{U2}, T_{U3}, T_{U5}\}$

& the set of down time is

$S_f = \{T_{d1}, T_{d2}, T_{d3}\}$

The history of this generator is shown in Fig.2

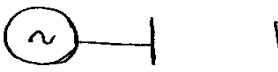
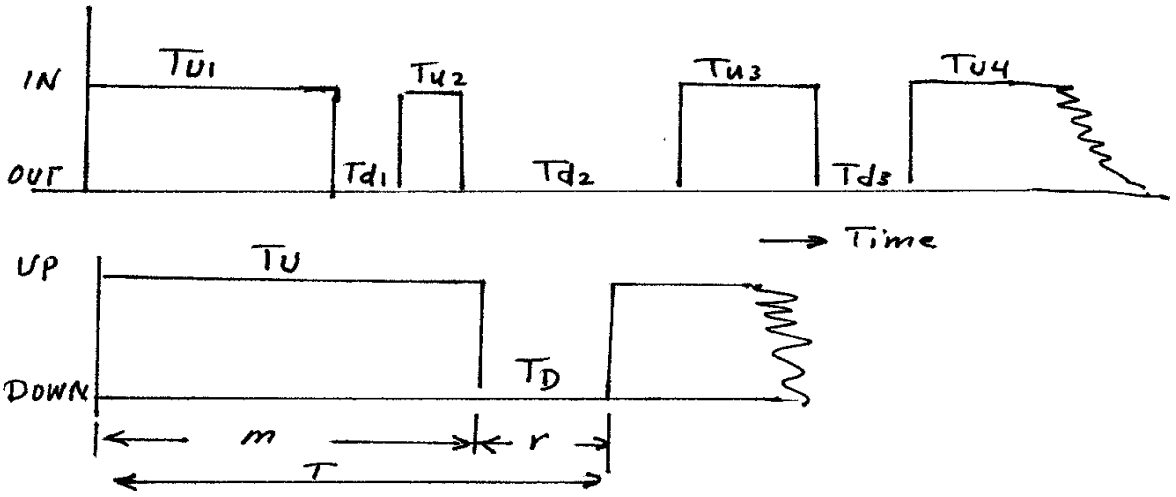


Fig.1



$$\text{Mean Service time} = T_U = \sum_{i=1}^{n} \frac{T_{Ui}}{n_u} = m$$

$$\text{Mean repaire time} = T_D = \sum_{i=1}^{n} \frac{T_{Di}}{n_d} = r$$

Let: m = mean time to failure $= \frac{1}{\lambda}$ $= MTTF$

r = mean time to repair $= \frac{1}{\mu}$ $= MTTR$

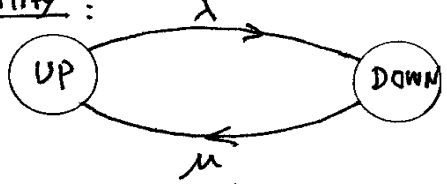
$T = m + r$ = mean time between failure $= \frac{1}{f}$

f = cycle frequency and T = cycle time.

$$\frac{\Sigma T_{Di}}{\Sigma T_{Ui} + \Sigma T_{Di}} = FOR = \frac{r}{m+r} = \frac{1/\mu}{1/\lambda + 1/\mu} = \frac{\lambda}{\lambda+\mu}$$

$$\text{Availability} = A = \frac{m}{m+r} = \frac{1/\lambda}{1/\lambda + 1/\mu} = \frac{\mu}{\lambda+\mu}$$

State probability:



Markov Model of one component

Generation System Reliability Evaluation

In all power system studies, the time period is devided into two periods:

1. Planning period
2. Operation period

- In generation system reliability studies same subdivision is made; i.e
  1. For planning period, we have:
     - Static generating capacity reliability evalnation
  2. For operation period, we hoave:
     - Spinining generating capacity reliability evalnation (spining reserve)

- What is RESERVE?
  - In static studies: reserve must be provided for the following reasons:
    * Scheduled or maintenance outages.
    * Forced, unplanned or unscheduled outages.
    * Load groth in excess of expectation.
  - In Spining Studies: reserve must be provided for:
    * Forced outages.
    * Load forcast uncertinities.

Note: In this set of notes only static evaluation will be considered.

Static Generating Capacity Reliability evaluation:

There are two methods in static capacity Reliability evaluation:

1. The deterministic (non-probabilistic) method.
2. The probabilistic approach.

- The deterministic (non-probabilistic) method:

In the deterministic method, there are two most common rule-of-thumb or non-probabilistic methods:

(a) The percentage reserve margin
(b) One additional largest unit.

These two methods do not account for differences in system size, different Load characteristics or the effect of different size and type of units.

In consequence two systems having the same percentage reserve can have vastly different probabilistic risk, i.e probability of not meating the load.

- In both deterministic and probabilistic methods, the outage probability of the capacities must be constructed first.

Capacity outage probability table

This table is also known as the generation model. It is constructed using Binomial distribution. Although the Binomial distribution has limited applications when the unit sizes and FOR's are differs, but it can be applied in such cases by starting with the smallest unit and continue to add one unit at a time to the table untill all units have been processed. These concepts can be illustrated in the following example:

Ex. Construct the capacity outage probability table for the following system:

The system consists of: 2 X 3 MW Units with FOR = 0.02

and 1 X 5 MW Units with FOR = 0.02

Solution: Consider first the 2 X 3 MW Units:

For these two units, the individual probability table is:

Table.1 ($P = 1 - 0.02 = 0.98$)($q = 0.02$)

Capacity out of Service		Probability
P_1P_2	0 MW	0.9604
$P_1q_2 + P_2q_1$	3 MW	0.0392
q_1q_2	6 MW	0.0004
		1.0000

- 5 MW unit in service ($q_3 = 0.02$, $P_3 = 0.98$):

Table.2

Capacity out	Probability
0+0 = 0 MW	(0.9604)(0.98) = 0.941192
3+0 = 3 MW	(0.392)(0.98) = 0.038416
6+0 = 6 MW	(0.0004)(0.98) = 0.000392
	0.980000

- Unit 5MW out of service :

Table.3: 5MW out of service $q_3 = 0.02$

Capacity out	Probability
0+5 = 5MW	(0.9604)(0.02) = 0.019208
3+5 = 8 MW	(0.0392)(0.02) = 0.000784
6+5 = 11 MW	(0.0004)(0.02) = 0.000008
	0.020000

The Final table 4 can be found by combining Table.2 & Table.3 as:

Table.4 Capacity outage probability table for the three Units

Capacity out of service (MW)	Individual probability	Commulative Probability
0	0.941192	1.000000
3	0.038416	0.058808
5	0.019208	0.020392
6	0.000392	0.001184
8	0.000784	0.000792
11	0.000008	0.000008
	1.000000	

- Note the tables above can be truncated by omitting all capacity outages for which the probabilities are less than 10^{-6} or 10^{-8}.

1. Deterministic Method:

To illustrate the application of the deterministic method, consider the following four generating systems:

System 1 - 24x10 MW	Units having a forced outage rate of 0.01
System 2 - 12x20 MW	Units having a forced outage rate of 0.01
System 3 - 12x20 MW	Units having a forced outage rate of 0.03
System 4 - 22x10 MW	Units having a forced outage rate of 0.01

In each of these systems the units are identical and therefore, the binomial distribution can be used to evaluate the capacity outage probability tables.

From the binomial distribution, the probability of given capacity on forced outage or unavailability is given by:

$$P_r(x=r, n, q) = C_r^n \, p^{n-r} q^r$$

where $C_r^n = \frac{n!}{r!(n-r)!}$

n = number of units

r = number of units unavailable (on forced outag)

p = availability (Probability of having in-service)

q = Unavailability (forced outage rate)

Consider System-1 as an example, the probability of 30 MW out of service is

$$P_3 = \frac{24!}{21!\,3!} \times (0.99)^{21} (0.01)^3$$

$$= 0.001639 \quad \text{(to six decimal places)}$$

Using this technique for all systems, the following outag probability tables can be evaluated with truncation of the table to 10^{-6}.

Table 1 Capacity Outage Probability Tables for systems 1–4

System 1 Capacity (MW)		*Probability*	
Out	*In*	*Individual*	*Cumulative*
0	240	0.785678	1.000000
10	230	0.190467	0.214322
20	220	0.022125	0.023855
30	210	0.001639	0.001730
40	200	0.000087	0.000091
50	190	0.000004	0.000004

System 2 Capacity (MW)		*Probability*	
Out	*In*	*Individual*	*Cumulative*
0	240	0.886384	1.000000
20	220	0.107441	0.113616
40	200	0.005969	0.006175
60	180	0.000201	0.000206
80	160	0.000005	0.000005

System 3 Capacity (MW)		*Probability*	
Out	*In*	*Individual*	*Cumulative*
0	240	0.693841	1.000000
20	220	0.257509	0.306159
40	200	0.043803	0.048650
60	180	0.004516	0.004847
80	160	0.000314	0.000331
100	140	0.000016	0.000017
120	120	0.000001	0.000001

System 4 Capacity (MW)		*Probability*	
Out	*In*	*Individual*	*Cumulative*
0	220	0.801631	1.000000
10	210	0.178140	0.198369
20	200	0.018894	0.020229
30	190	0.001272	0.001335
40	180	0.000061	0.000063
50	170	0.000002	0.000002

Comparison between the two deterministic methods

(a) *Percentage reserve margin*

Assume that the expected load demands in systems 1, 2, 3 and 4 are 200, 200, 200 and 183 MW respectively. The installed capacity in all four cases is such that there is a 20% reserve margin, i.e. a constant for all four systems. The probabilistic or true risks in each of the four systems can be found from Table 1 and are:

risk in system 1 = 0.000004
risk in system 2 = 0.000206
risk in system 3 = 0.004847
risk in system 4 = 0.000063

These values of risk show that the true risk in system 3 is 1000 times greater than that in system 1. A detailed analysis of the four systems will show that the variation in true risk depends upon the forced outage rate, number of units and load demand. The percentage reserve method cannot account for these factors and therefore, although using a 'constant' risk criterion, does not give a consistent risk assessment of the system.

(b) *Largest unit reserve*

Assume now that the expected load demands in systems 1, 2, 3 and 4 are 230, 220, 220 and 210 MW respectively. The installed capacity in all four cases is such that the reserve is equal to the largest unit which again is a constant for all the systems. In this case the probabilistic risks are:

risk in system 1 = 0.023855
risk in system 2 = 0.006175
risk in system 3 = 0.048650
risk in system 4 = 0.020229

The variation in risk is much smaller in this case, which gives some credence to the criterion. The ratio between the smallest and greatest risk levels is now 8:1 and the risk merit order has changed from system 3–2–4–1 in the case of 'percentage reserve' to 3–1–4–2 in the case of the 'largest unit' criterion.

It is seen from these comparisons that the use of deterministic or 'rule-of-thumb' criteria can lead to very divergent probabilistic risks even for systems that are very similar. They are therefore inconsistent, unreliable and subjective methods for reserve margin planning.

14. Load Modelling

Load Demand Representations (Load Model)

For reliability studies, the production of power depends on the load variations on the generating units during the year. Consequently, one should develope an accurate time load-model representation to facilitate reliability analysis.

It is found that the 8760 hourly loads per year model (Annual load curve) is the most accurate representation. But the data required is too large. However, in less detailed simulation, a daily load-duration curve as illustrated in Fig. 2 is used.

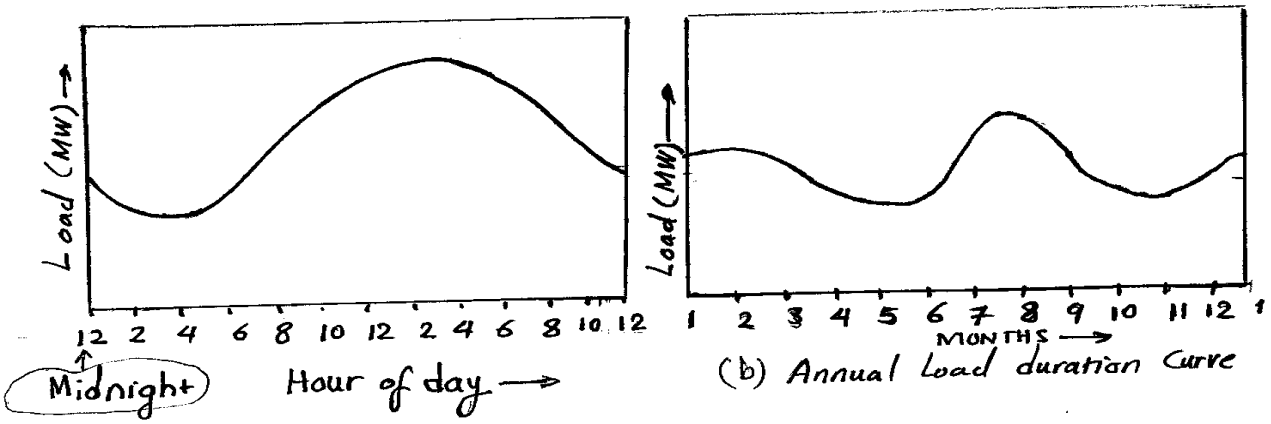

Load (MW)
12 2 4 6 8 10 12 2 4 6 8 10 12
Midnight
Hour of day
(a) Typical daily load variation curve
Load (MW)
1 2 3 4 5 6 7 8 9 10 11 12 1
MONTHS
(b) Annual load duration curve

Fig. 1

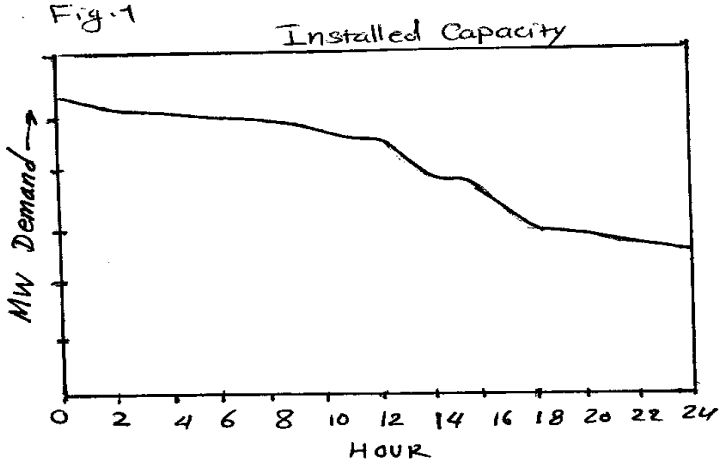


Fig. 2
Daily load-duration Curve

Construction of the load Model

For understanding the construction of the load model, it is necessary that engineer must be familiar with the following important terms:

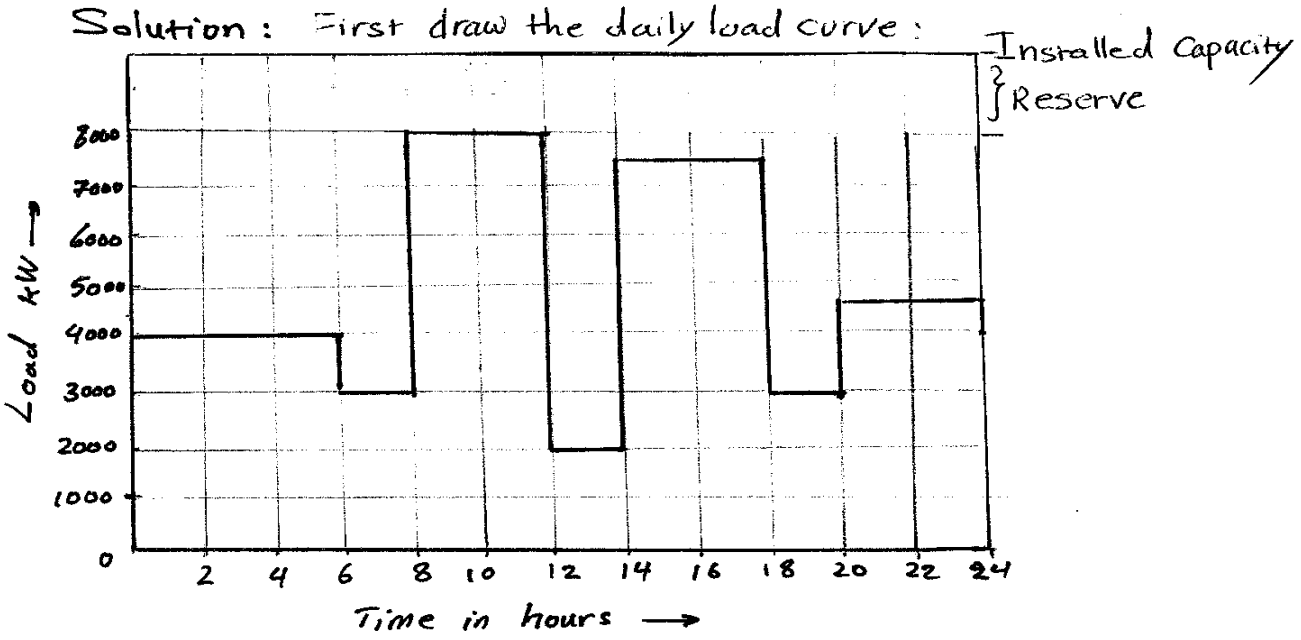
1. Capacity of the plant = Sum of all generating Units.
   Also called the installed capacity.
2. Maximum demand = Maximum load occurs on the plant.
3. Energy generated : is the sum of all power generated during a specified period of time.
4. Average load $= \frac{\text{Energy generated}}{\text{Time}}$
5. Load factor $= \frac{\text{Average Load}}{\text{Maximum Demand}}$.
6. Capacity (or plant) factor $= \frac{\text{Average load}}{\text{Capacity of the plant}}$.
8. Load Duration Curve : is the frequency of occurance of the peak loads during a specified duration in descending order.

Example 1. A generating station has the following daily loads:

0 to 6 hr	4,000 kW
6 to 8 hr	3,000 kW
8 to 12 hr	8,000 kW
12 to 14 hr	2,000 kW
14 to 18 hr	7,500 kW
18 to 20 hr	3,000 kW
20 to 24 hr	4,500 kW

Sketch the load duration curve and determine the load factor and plant capacity factor assuming the capacity of the plant 11000 kW.

Solution: First draw the daily load curve:



Daily Load Curve

Capacity of the plant = 11,000 kW.

Maximum demand of the plant = 8,000 kW

generated in 24 hours:

$$= 4,000 \times 6 + 3,000 \times 4 + 8,000 \times 4 + 2,000 \times 2 + 7,500 \times 4 + 4,500 \times 4$$

$$= 120,000 \text{ kWh}.$$

3

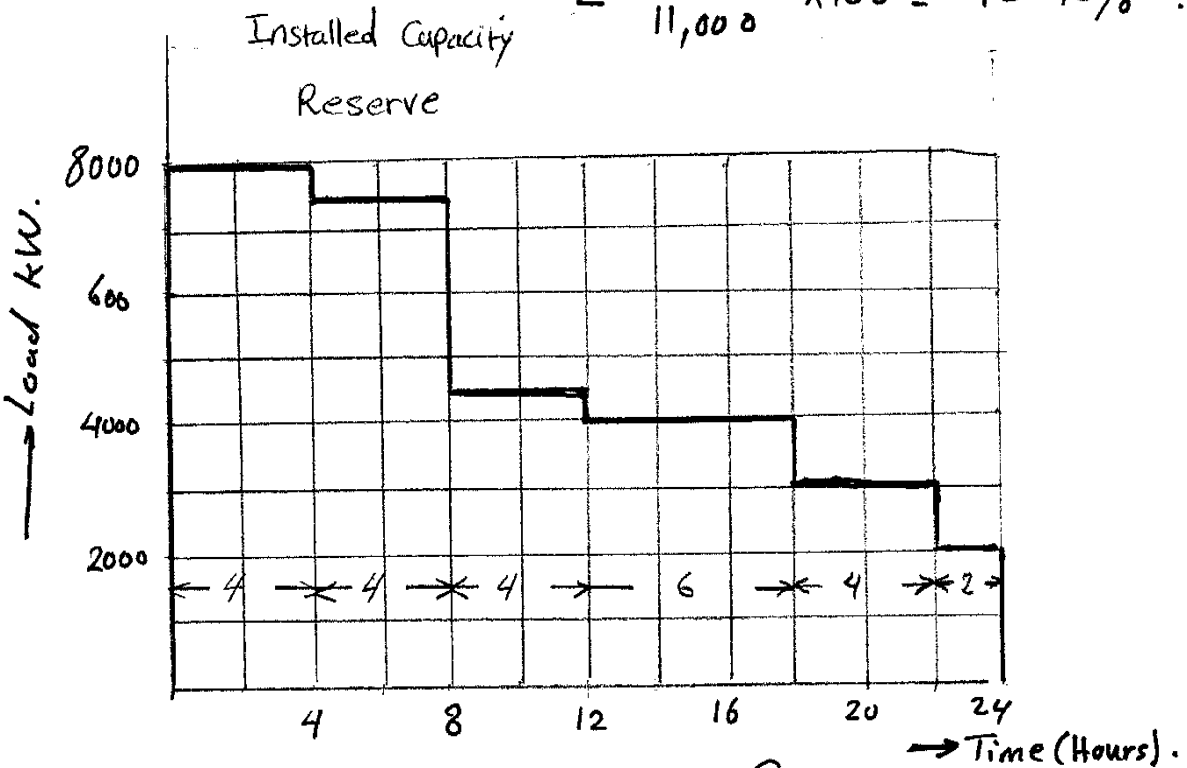
$$\text{Average load} = \frac{\text{kWh generated}}{\text{Time in hours}} = \frac{120,000}{24} = 5,000 \text{ kW}.$$

$$\text{Load Factor} = \frac{\text{Average Load}}{\text{Maximum Demand}} \times 100$$

$$= \frac{5,000}{8,000} \times 100 = 62.5\% .$$

$$\text{plant capacity factor} = \frac{\text{Average load (demand)}}{\text{Rated Capacity of power plant}}$$

$$= \frac{5,000}{11,000} \times 100 = 45.45\% .$$



Load Duration Curve.

15. Generation System Reliability Evaluation

2. Probabilistic Method

Loss of Load probability Method (LOLP)

Earlier, the capacity outage probability tables were used to evaluate the probabilistic risk assuming constant load level. In practice, however, loads vary and therefore a **load model** as well as a **generation model** are required.

The loss of load probability method is probably the most widely accepted and used of all methods for evaluating the risk level in generating systems. The load model used in this method is the load characteristic shown in Fig.1 below.

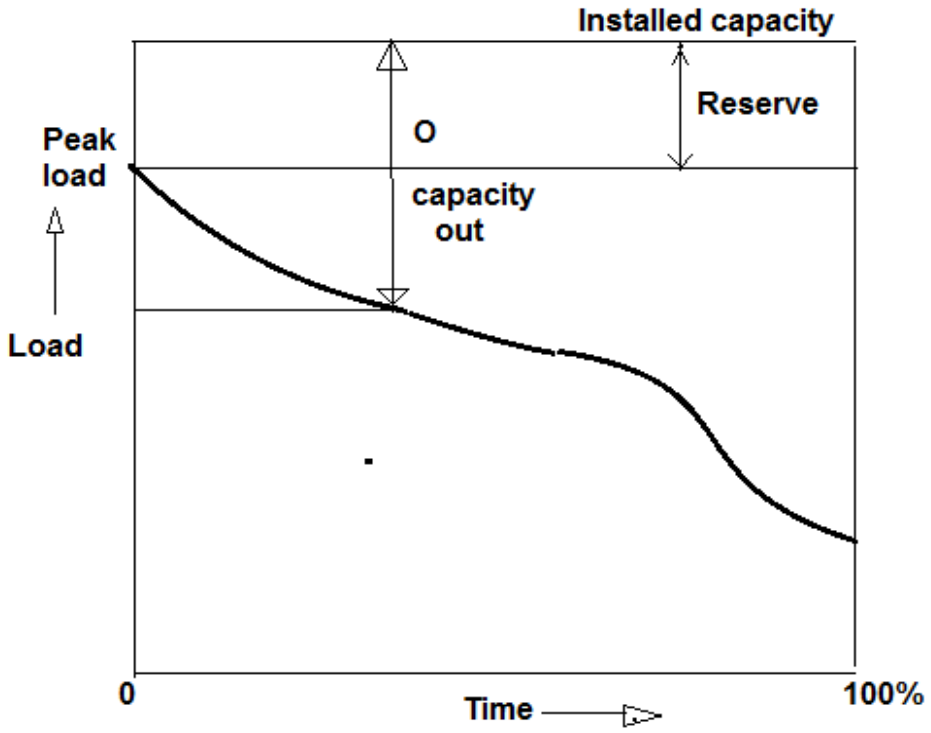


Fig.1

This load characteristic is the load duration curve (LDC) that had been discussed previously in lecture no.14. It is a plot of the load as a function of the number of time units which a load exceeded the indicated value. It may represent a daily peaks loads in a year , the hourly peak loads in a day or any other appropriate time interval of interest ,e.g. deep summer period in a middle east country with a time duration of ,say, 3 months . Similarly it may or

may not includes all the days in that period , for instant , weekends may be neglected since loss of load is very improbable during these periods.

For simplicity in the following examples we will consider the load characteristic to be a plot of daily peak load in a year and therefore the abscissa axis represents 365 days.

It is clear from the load characteristic that a capacity outage less than the reserve will not cause a loss of load. Consider now:

O_i – the i - th outage in the system outage probability table.

P_i – the probability of this i – th outage.

t_i – the number of time units which this outage causes loss of load .

Then the contribution to the system loss of load by outage **O_i** is **Pi t_i** time units. Therefore, the total expected loss of load for the time interval being studied is

$$E(t) = \sum_{i=1}^{n} \mathrm{P_i}\ t_i \qquad \text{time units}$$

For which, if we consider the load characteristic is the daily peak load variation over a year, the units are expected loss of load in days per year.

To illustrate these principles consider a system having 5x60 MW units each with FOR of 0.03. For simplicity, consider the load characteristic to be linear with a peak load of 240MW and a low load of 100Mw as shown in Fig.2.

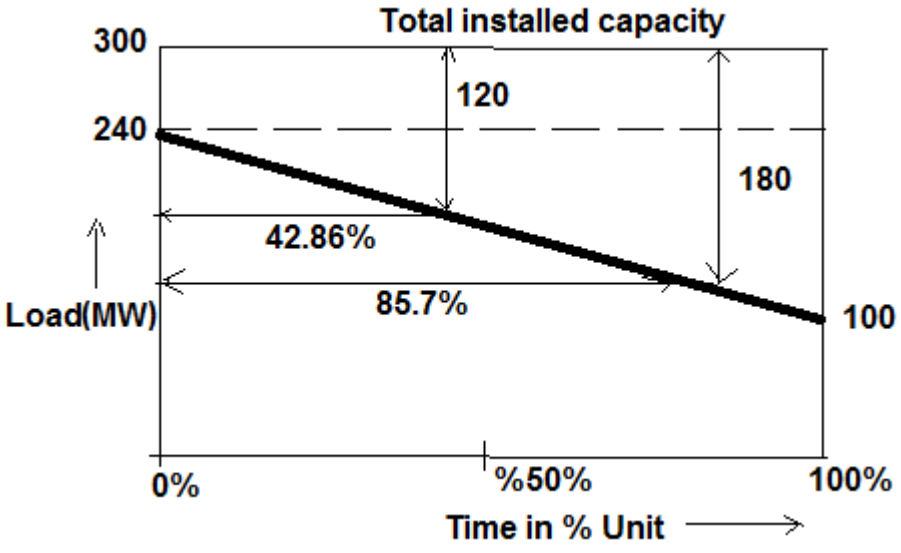


Fig.2

The table of capacity outage probability (generation model) and expected loss of load is therefore:

Capacity out (MW)	Individual probability (P_i)	Time t_i %	Expected Load loss $[P_i * t_i]$
0	0.858734	0	-----
60	0.132794	0	-----
120	0.008214	42.86	0.352052
180	0.000254	85.71	0.021770
240	0.000004	100.00	0.000400
	1.000000		$\sum P_i t_i = 0.374222\%$

Expected loss of load if 100% = 365 days is

$$E(t) = 0.374222 \times (365 /100) = 1.365910 \text{ days/year}$$

This value would reasonably be called excessive. A value often considered as reasonable is **0.1** days /year.

- **Effect of forced outage rate variation**

Now we shall consider the effect of changing the value of FOR on the loss of load by considering the same system but each unit having a FOR of 0.01. The tables are therefore:

Capacity out (MW)	Individual probability (P_i)	Time t_i %	Expected Load loss $[P_i * t_i]$
0	0.950990	0	-----
60	0.048030	0	-----
120	0.000970	42.86	0.041574
180	0.000010	85.71	0.000857
240	0.000000	100.00	0.000000
	1.000000		$\sum P_i t_i = 0.042431\%$

- **Effect of peak load variation**

If the same generating system is analysed for its ability to feed a load characteristic having the same load factor used previously but having peak

loads between 120MW and 300MW, then the following expected loss of loads would be obtained.

Peak Load (MW)	Loss of Load D/y	Peak Load (MW)	Loss of Load D/y
300	7.820810	200	0.052679
280	5.265140	180	0.001564
260	2.712127	160	0.001043
240	0.154874	140	0.000522
220	0.103754	120	0.000000

If a risk level of 0.1 days/year is considered acceptable, then the above generating system is capable of supplying a load characteristic of the assumed shape having a peak load of 220MW. This is known as the load carrying capability of the generating system. A plot of the previous table showing loss of load in days /year as a function of peak load is shown in Fig.3.

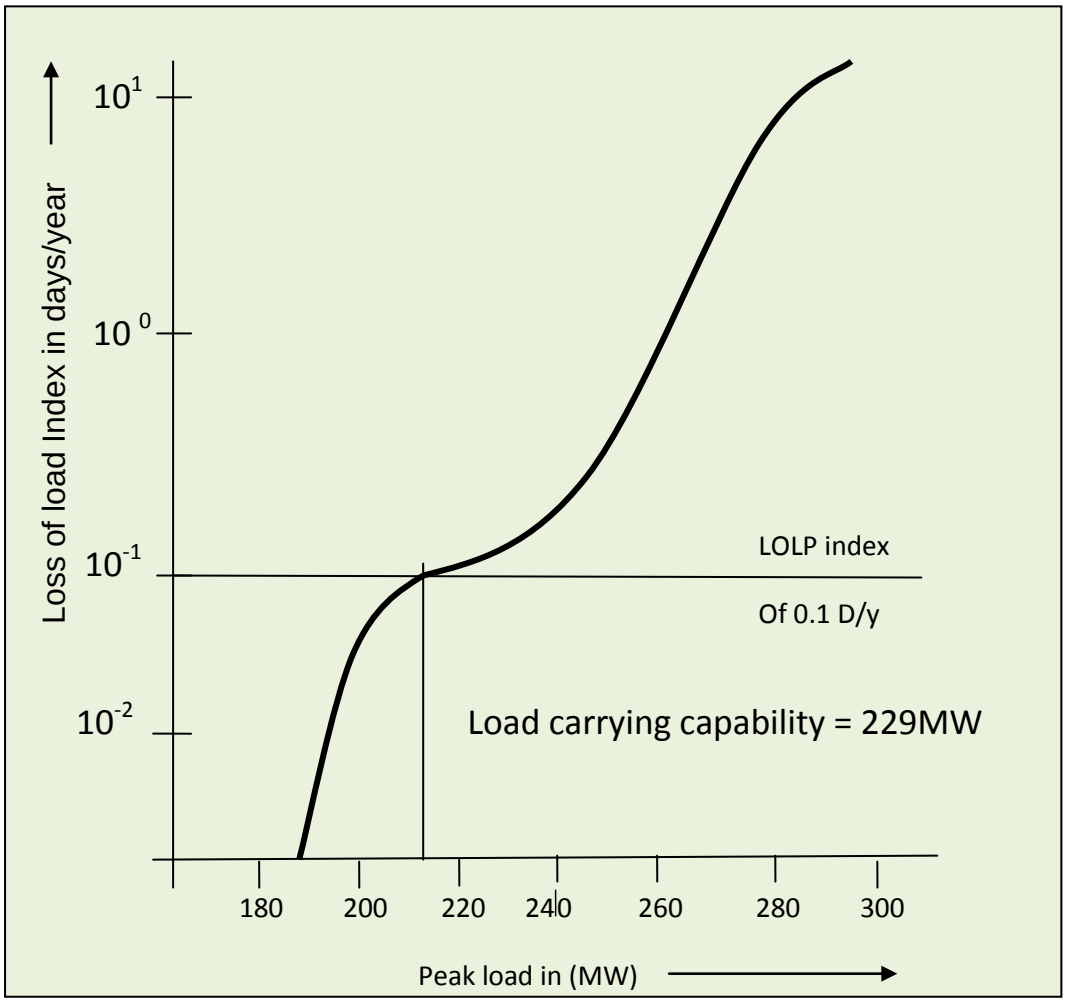


Fig.5

Expansion Study

Because it takes a considerable period of time for a generating unit to be commissioned after deciding it becomes necessary, it is essential to predict the need for it by using expansion planning studies. The loss of load probability method can be extremely valuable in such studies.

To do this, the expected load growth must be known, the accepted risk level must be determined and the size of unit to be added must be decided upon in advance.

To illustrate the technique consider an acceptable risk level of 0.1 days/year and annual growth rate of 10% .Also consider that in year 1 of study the peak is 220MW supplied by the previous generating system of 5x60MW units each having a FOR of 0.01. Also assume that the shape of the load characteristic remains unchanged and is given by the previous linear characteristic.

If the period of study is 8 years, the peak load in each year will be:

Year	1	2	3	4	5	6	7	8
Peak load (MW)	220	242	266	293	322	354	390	429

Using the previous technique to deduce the expected loss of load in each year of study, the following expansion plan would be attained if 60 MW units are added.

Year	Generating system (MW)	Loss of load (D/year)	Comment
1	5x60	0.10	
2	5x60	0.15	New unit to be added
	6x60	0.01	
3	6x60	0.09	
4	6x60	0.19	New unit to be added
	7x60	0.01	
5	7x60	0.11	New unit to be added
	8x60	0.01	
6	8x60	0.01	
7	8x60	0.23	New unit to be added
	9x60	0.01	
8	9x60	0.1	

It is evident therefore that for the system being considered , a 60 MW unit should be added in years 2,4,5, and 7 , but the system can remain unchanged in years 1,3,6, and 8.

Clearly this expansion study cannot be done in isolation and a present worth cost analysis would need to be done to determine the optimum expansion plan. However, this type of analysis should form an integral part of the planning exercise.

Inclusion of maintenance

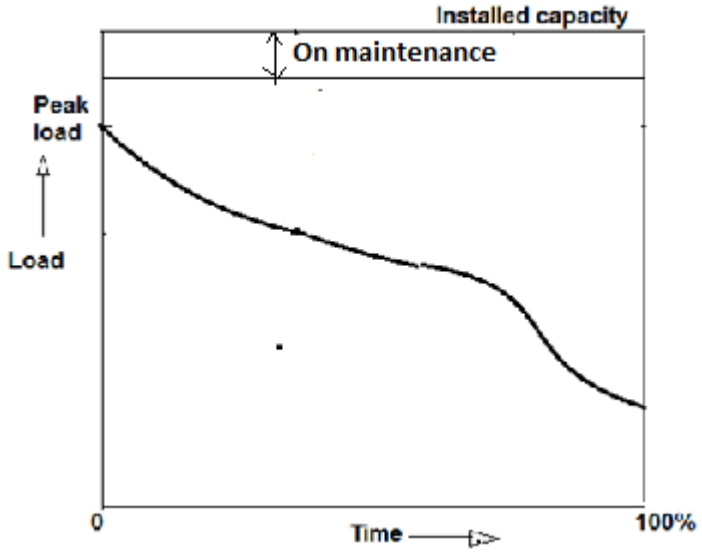
So far we have assumed that the generating system remains constant throughout the year. This is clearly not the case since units will be removed for scheduled maintenance. There are several ways in which this can be achieved; only one of which is the exact method.

The exact method involving modifying the capacity outage probability table so that the true generating system is modelled This can be time consuming although simplified if units are removed from the table as described previously rather than building a new table with these units involved.

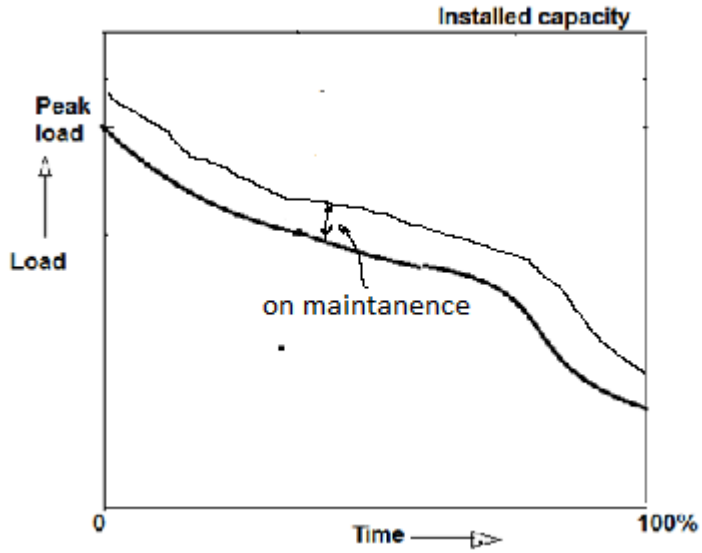
To simplify, two procedures may be used instead of modifying the table, the first is to subtract the capacity on maintenance from the installed capacity and therefore reduce the reserve without changing the capacity probability table. The second is to add the capacity on maintenance to the load. If maintenance is done only during the low load period and the load characteristic is for the whole year, the capacity on maintenance is added only to the low load period. These techniques are shown below in schematic form in Fig.6.

It is clear that the last two methods will be approximate although the errors can be negligable if the capacity on maintanance is very small compared with the installed capacity . The metods give calculated risks that are greater than the true risk which is increases as the prcentage of capacity on outage increases .

(a)


Installed capacity
On maintenance
Peak load
Load
0
Time
100%

(b)


Installed capacity
Peak load
Load
on maintanence
0
Time
100%

(b)

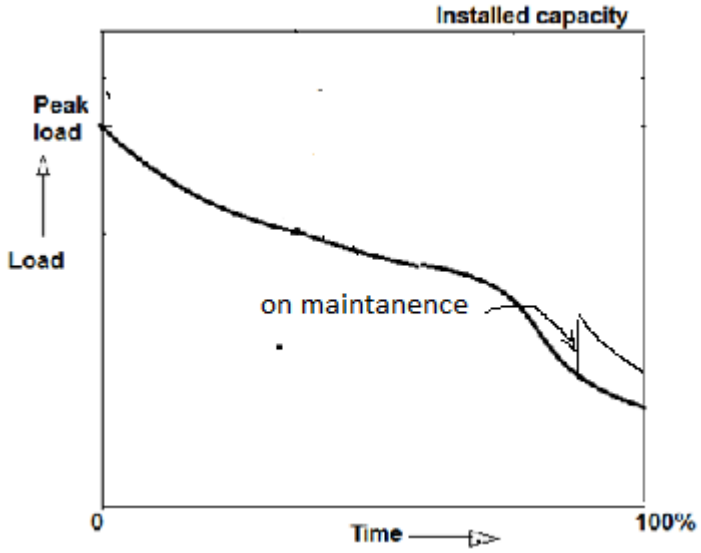

Installed capacity
Peak load
Load
on maintanence
0
Time
100%

Fig.6

Load Forecast Uncertainties

So far we have assumed that the load on the system is known exactly. In planning studies this is not the case and the load must be forecasted or predicted from previous experience. It has been suggested that the distribution of forecast uncertainties may be represented by normal distribution and this will be assumed in the following discussion.

A normal distribution can be divided into a number of discrete intervals, each giving its probability of occurrence of the mid-point of that interval .If the mean value of the normal distribution represents the forecasted load , then the probability that this forecasted load will be in error can be deduced from the probability of these intervals.

To show the application of this technique, we will divide the normal distribution seven intervals, the mid-points of which are -3σ, - 2σ, - 1 σ, μ, +1 σ, +2 σ, and +3 σ where σ is the standard deviation and μ is the mean.

The probabilities of these intervals can be found from table or calculated from the normal distribution equation. They are shown in the following sketch (Fig.7):

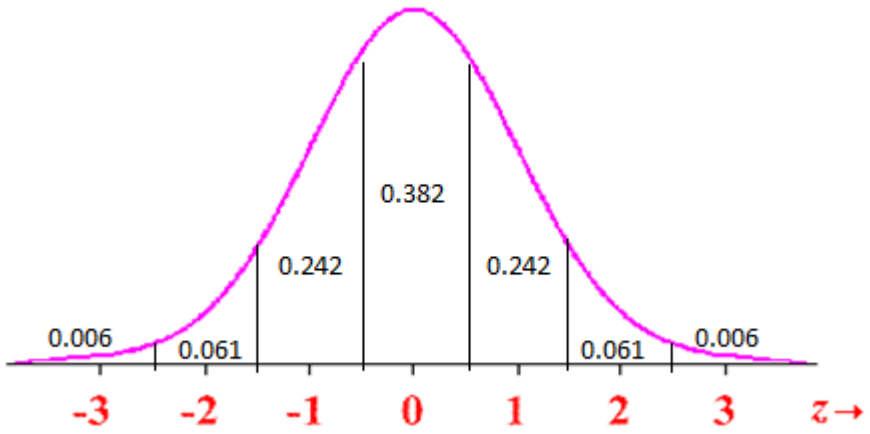


Fig. 7 Number of standard deviations from mean (mean = forecasted load).

If the forecasted load is used to evaluate the LOLP , then the calculated risk level will be optimistic since it is possible for the load to be greater than this forecasted load and the system would be at risk.

Dividing the normal distribution curve into seven steps indicates that the forecasted load has a probability of occurrence of only 0.382.Therefore the LOLP oversited with this load level also only has a probability of occurrence of 0.382.To evaluate the true risk of the system, the LOLP associate with each discrete load level must be evaluated, weighted by its probability of occurrence and all contributions summed together.

To illustrate this consider the previous 5x60 MW generating system each having a FOR of 0.01 supplying a forecasted peak load of 200 MW.Previously ,it was deduced that the expected load loss was 0.052679 D/yr.

Consider now that this forecasted load is a mean value of a normal distribution with a 10% uncertainty (≡ standard deviation). Using the previous technique to evaluate the LOLP for each of the seven possible load levels gives the following table:

Number of standard deviation	Load level (MW)	probability of load level	Expected load loss(d/yr)	weighted LOLP (col.3 x col.4)
- 3	140	0.006	0.000522	0.000003
- 2	160	0.061	0.001043	0.000064
- 1	180	0.242	0.001564	0.000378
0	200	0.382	0.052679	0.020123
+ 1	220	0.242	0.103759	0.025110
+ 2	240	0.061	0.154874	0.009447
+ 3	260	0.006	2.711127	0.016267
				0.071392 D/yr

Note: the values in column 4 have been deduced previously on page 4.

It is evident from this table that the LOLP has increased.

16. Transmission System Reliability

Power System Reliability Evaluation Dr.Mohammed Tawfeeq
Transmission System Reliability Evaluation

The transmission system, as a part of a power system are widely branched, has great length, operates under variable weather conditions, and is subjected to the action of atmospheric disturbances. Hence, it is important to consider their reliabilities under normal and stormy weather conditions.

In general their are two methods used in transmission system reliability evaluation.

1. Markov technique (Exact method).
2. Frequency and duration method (Approximate method).

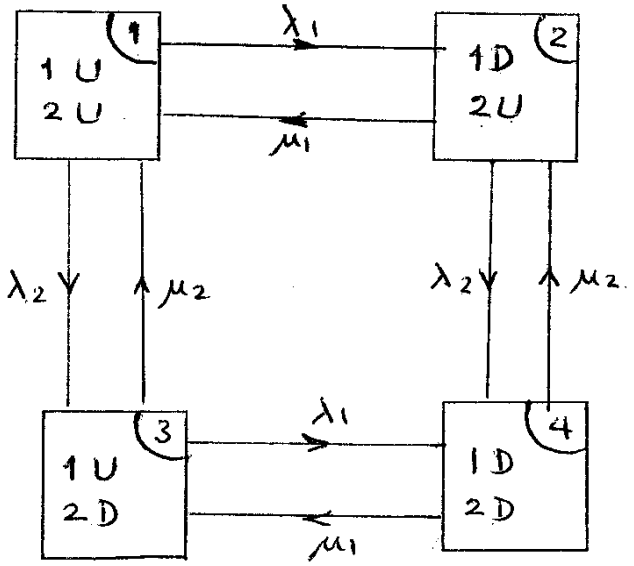
- The Markov technique which is based on the state concepts and stochastic processes gives an accurate method for reliability evaluation for such system.
- A more simple and easy method is the frequency and duration method in which the expected outage frequency and outage duration at any point in any transmission system can be calculated. In this technique the following assumption is made:
  1. Each component must be in either of two states: UP (available) or down (unavailable).
  2. For a series arrangement, power is to flow when all components are in the up state.
  3. For a parallel arrangement, power is to flow only when one or more parallel paths are to exist.

Transmission System Reliability Evaluation

Markov Modeling:

State probabilities of two components (Markov Modeling) & the frequency and duration approach:

The state space transition diagram for a two components system is (According to Markov modeling)



If the probability of a component being in the down state is $\frac{\lambda}{\lambda+\mu}$ and in the up state is $\frac{\mu}{\lambda+\mu}$, then:

Let:

P_1 = Probability of being in state 1 = $\frac{\mu_1}{\mu_1+\lambda_1} \cdot \frac{\mu_2}{\mu_2+\lambda_2}$

P_2 = probability of being in state 2 = $\frac{\lambda_1}{\mu_1+\lambda_1} \cdot \frac{\mu_2}{\mu_2+\lambda_2}$

P_3 = probability of being in state 3 = $\frac{\mu_1}{\mu_1+\lambda_1} \cdot \frac{\lambda_2}{\mu_2+\lambda_2}$

P_4 = probability of being in state 4 = $\frac{\lambda_1}{\mu_1+\lambda_1} \cdot \frac{\lambda_2}{\mu_2+\lambda_2}$

probability of down state = $P_2+P_3+P_4$.

Frequency & Duration

Now define :

1. Frequency of encountering state = probability of being in state X × rate of departure

or

$$f_1 = P_1 (\lambda_1 + \lambda_2) \quad \text{◊/yr}$$
$$f_2 = P_2 (\lambda_2 + \mu_1) \quad \text{◊/yr}$$
$$f_3 = P_3 (\lambda_1 + \mu_2) \quad \text{◊/yr}$$
$$f_4 = P_4 (\mu_1 + \mu_2) \quad \text{◊/yr}$$

f(down state) = f(up state) = f_1.

2. Duration in state = $\frac{\text{Probability of encountering state}}{\text{Frequency of encountering state}}$

∴ Duration (state 1) = $\frac{P_1}{f_1}$

Duration (state 2) = $\frac{P_2}{f_2}$

Duration (state 3) = $\frac{P_3}{f_3}$

Duration in (state 4) = $\frac{P_4}{f_4}$

Duration in the (down state) = $\frac{P(\text{down state})}{F(\text{down state})}$

3. Definitions:

λ = Expected failure rate of the system

r = Average duration of the outage.

U = Unavailability or annual outage time

Approximate Method

Series and Parallel Components or lines

1. Series component

For the series components, both must be (up) for the system to be successful, i.e., the up state is tate 1 in the state space diagram.

Hence for state 1:

$$A_s = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\mu_s}{\lambda_s+\mu_s} \quad \cdots (1)$$

Assuming exponential distribution:

Probability of succes (State 1) $= e^{-\lambda_1 t} \cdot e^{-\lambda_1 t} = e^{-\lambda_s t}$.

$$= e^{-(\lambda_1+\lambda_2)t} = e^{-\lambda_s t}.$$

$$\simeq (\lambda_1+\lambda_2)t = \lambda_s t.$$

$$i.e \quad \lambda_s = \lambda_1+\lambda_2 = \Sigma\lambda \quad \cdots (2)$$

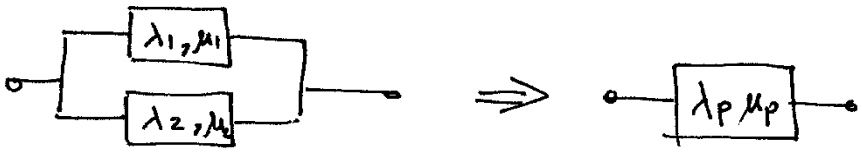
Now substituting $\mu = \frac{1}{r}$ in equation (1) and Solve for r_s yield:

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1\lambda_2 r_1 r_2}{\lambda_s}$$

since $\lambda_1\lambda_2 r_1 r_2$ is very small

$$\text{Hence } r_s = \frac{\sum_{i=1}^{n} \lambda_i r_i}{\lambda_s} = \frac{\Sigma\lambda r}{\lambda_s} = \frac{\Sigma\lambda r}{\Sigma\lambda} \quad \cdots (3)$$

2. Parallel Components



If the components are in parallel, both must fail for system failure, i.e the down state is state 4 of the above state space diagram.

$$\therefore \quad \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\lambda_p}{\lambda_p+\mu_p} = U_p \quad (4)$$

Again if we assume exponential distribution

probability of failure $= e^{-\mu_1 t} \cdot e^{-\mu_2 t} = e^{-\mu_p t}$.

$$\simeq \mu_1+\mu_2 = \mu_p .$$

or $$\frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\therefore \quad r_p = \frac{r_1 r_2}{r_1+r_2} \quad \ldots\ldots \quad (5)$$

Solving for λ_p from eq. (4)

$$\lambda_p = \frac{\lambda_1\lambda_2(r_1+r_2)}{1+\underbrace{\lambda_1 r_1+\lambda_2 r_2}_{\text{very small}}}$$

$$\therefore \quad \lambda_p = \lambda_1\lambda_2(r_1+r_2)$$

or $$U_p = \frac{\lambda_p}{\lambda_p+\frac{1}{r_p}} = \frac{\lambda_p r_p}{\underbrace{\lambda_p r_p}_{\text{small}}+1} \simeq \lambda_p r_p$$

$$= \lambda_1\lambda_2(r_1+r_2)\frac{r_1 r_2}{(r_1+r_2)}$$

$$\therefore \quad \boxed{U_p = \lambda_1\lambda_2 r_1 r_2} \quad (5)$$

Example: Consider the transmission system shown in Fig. A. The line reliability data is given in table 1. Calculate the load point (bus bar) reliability indices at bus bars B and C.

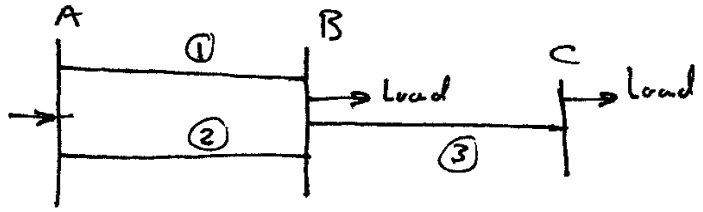


Table. 1

Line	λ (f/yr)	r (hr)
1	0.5	5
2	0.5	5
3	0.1	10

Solution: To find the indices at bus bar B, line 1 and 2 must be combined in parallel.

$$\lambda_B = \lambda_1\lambda_2(r_1+r_2)$$
$$= 0.5\times0.5(5+5)/8760 = 2.854\times10^{-4} \text{ f/yr.}$$

$$r_B = \frac{r_1 r_2}{r_1+r_2} = \frac{5\times5}{5+5} = 2.5 \text{ hr}$$

$$U_B = 2.854\times10^{-4}\times2.5/8760$$
$$= 8.145\times10^{-8} \text{ yrs/yr} = \text{probability}$$

or $U_B = 7.135\times10^{-4}$ hrs/yr.

To find the indices at bus bar C, line 1 and 2 must be combine in parallel (as above) and these combined with line 3 in series:

$$\lambda_C = \lambda_B + \lambda_3 = 2.854\times10^{-4} + 0.1 = 1.003\times10^{-1} \text{ f/yr.}$$
$$= 7.135\times10^{-4} + 0.1\times10$$

$$r_C = \frac{U_B + \lambda_3 r_3}{\lambda_C} \left(= \frac{\Sigma\lambda r}{\Sigma\lambda}\right) = 9.977 \text{ hr}$$
$$1.003\times10^{-1}$$

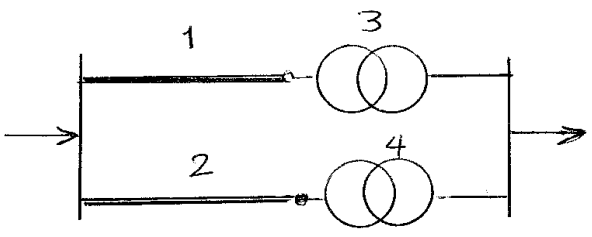
$$U_C = \lambda_C r_C = 1.001 \text{ hr/yr.} \quad (6)$$

17. Failure Mode Analysis

Failure Mode Analysis

The method used in the previous example is called the network reduction technique. In this method the critical or unreliable areas become absorbed into equivalent elements and become impossible to identify. The alternative is to inspect the system and compare a list of failure modes, i.e component outages, that must overlap to cause a system outage. These overlapping outages are effectively parallel elements and can be combined using the equations for parallel components. Any one of these overlapping outages will cause system failure and therefore, from reliability point of view, are effectively in series. The system indices can threfore be evaluated by applying the previous series equations to these overlapping outages.

As an example consider the following network



Component	λ(f/yr)	r(hr)
1	0.5	10
2	0.5	10
3	0.01	100
4	0.01	100

(a) Using network reduction

Combining element 1 and 3 in series gives:

$\lambda_{s_1} = \lambda_{1,3} = \lambda_1 + \lambda_3 = 0.5 + 0.01 = 0.51$ f/yr

$r_{s_1} = r_{1,3} = \frac{\lambda_1 r_1 + \lambda_3 r_3}{\lambda_1 + \lambda_3} = \frac{0.5 \times 10 + 0.01 \times 100}{0.51} = \frac{6}{0.51} = 11.76$ hr

$U_{1,3} = \lambda_s \times r_s = 0.51 \times 11.76 = 6$ hr/yr

The indices of components 2 and 4 combined will be identical:

$\lambda_{s_2} = \lambda_{2,4} = 0.51$ f/yr, $r_{2,4} = 11.76$ hr, $U_{2,4} = 6$ hr/yr.

∴ The indices for the load point:

$\lambda_p \simeq \lambda_{s1} \cdot \lambda_{s2} (r_{s1} + r_{s2})$

$= 0.51 \times 0.51 \left(\frac{11.76 + 11.76}{8760}\right) = \frac{0.26 \times 23.52}{8760}$

$= 6.98 \times 10^{-4}$ f/yr.

$r_p = \frac{r_{s1} r_{s2}}{r_{s1} + r_{s2}} = \frac{11.76 \times 11.76}{23.52} = \frac{138.24}{23.52} = 5.877$ hr

$U_p = \lambda_p r_p = 6.98 \times 10^{-4} \times 5.877 = 4.1025 \times 10^{-3}$ hr/yr.

(b) Using failure mode analysis:

For parallel system:

$\lambda_p = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{8760}$, $r_p = \frac{r_1 r_2}{r_1 + r_2}$, $U_p = \lambda_p r_p = \frac{\lambda_1 \lambda_2 r_1 r_2}{8760}$

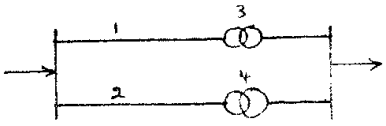
overlapping Outages	$\lambda_p = \frac{\lambda_1 \lambda_2 (r_1 + r_2)}{8760}$ λ (f/yr)	$r_p = \frac{r_1 r_2}{r_1 + r_2}$ r (hr)	$U_p = \lambda_p r_p$ U (hr/yr)
1 and 2	$\frac{0.5 \times 0.5 (10+10)}{8760} = 5.7 \times 10^{-4}$	5	2.854×10^{-3}
1 and 4	$\frac{0.5 \times 0.01 (100+10)}{8760} = 6.279 \times 10^{-5}$	9.091	5.708×10^{-4}
2 and 3	$\frac{0.5 \times 0.01 (110)}{8760} = 6.279 \times 10^{-5}$	9.01	5.708×10^{-4}
3 and 4	$\frac{0.01 \times 0.01 (200)}{8760} = 2.283 \times 10^{-4}$	50	1.142×10^{-4}
	$\lambda_s = \Sigma\lambda = 6.98 \times 10^{-4}$	5.88	4.102×10^{-3}
		$r_s = \frac{U_s}{\lambda_s}$	$= U_s = \Sigma U$

Although the second method is longer, it gives much more information. It clearly indicates that the failure rate and unavailability is mainly due to the overlapping failures of the two lines but that the average outage duration is mainly due to the overlapping outages of the two transformers. This information is necessary in assessing critical areas and deducing the area requiring investment, is not given by the network reduction technique.

Markov Techniques

As discussed previously the most precise method for evaluating the reliability of a network is Markov Processes. However it becomes very complicated even for relatively simple systems and the gain in precision has been shown to be negligible.

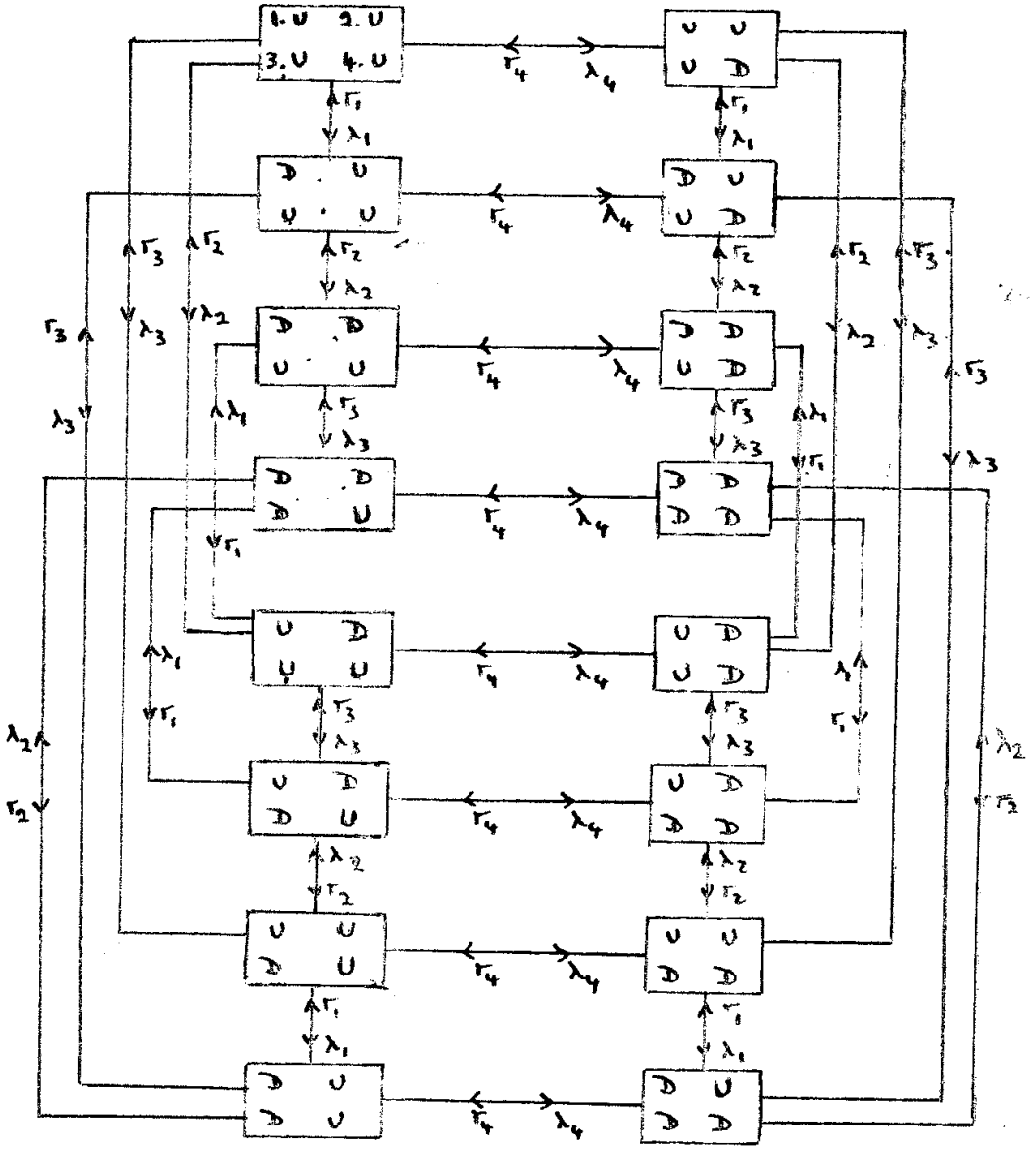
To illustrate the technique however consider the system used previously (page 8) and ignore maintenance effects and assume a single weather state.



If each component is considered to reside in one of two states (up or down), then for a 4-component system, there are $2^4 = 16$ states to consider. These are:

Component		1	2	3	4
States	1	U	U	U	U
	2	D	U	U	U
	3	U	D	U	U
	4	D	D	U	U
	5	U	U	D	U
	6	D	U	D	U
	7	U	D	D	U
	8	D	D	D	U
	9	U	U	U	D
	10	D	U	U	D
	11	U	D	U	D
	12	D	D	U	D
	13	U	U	D	D
	14	D	U	D	D
	15	U	D	D	D
	16	D	D	D	D

The state-space diagram for this would be :



This can then be solved using the techniques described previously.